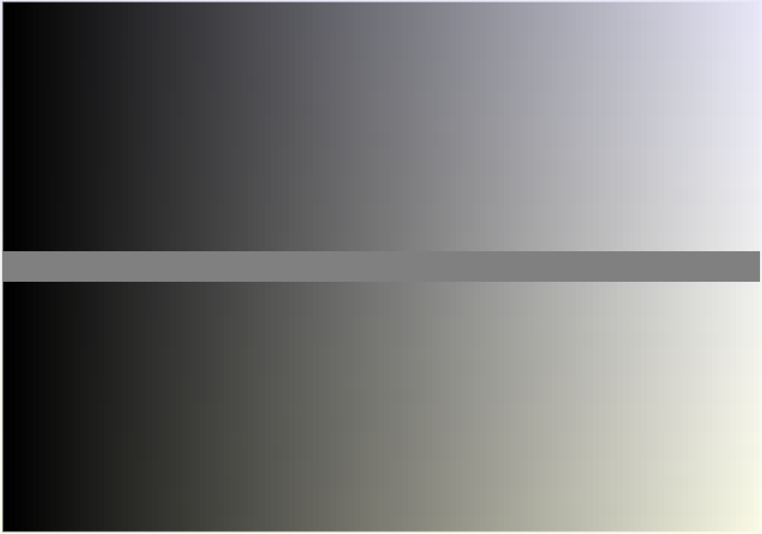
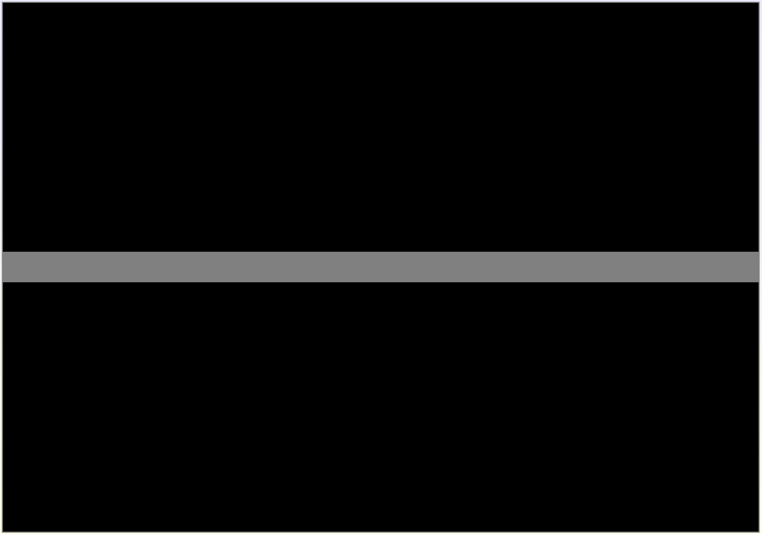
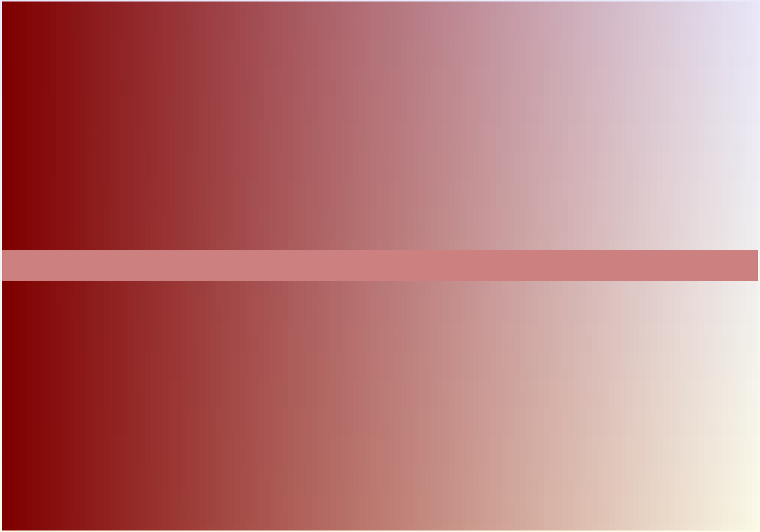
Gray Bar



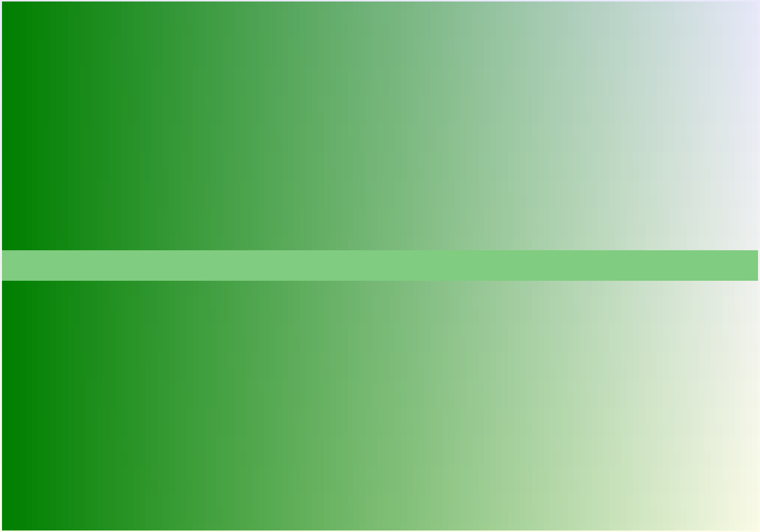
Gray Bar



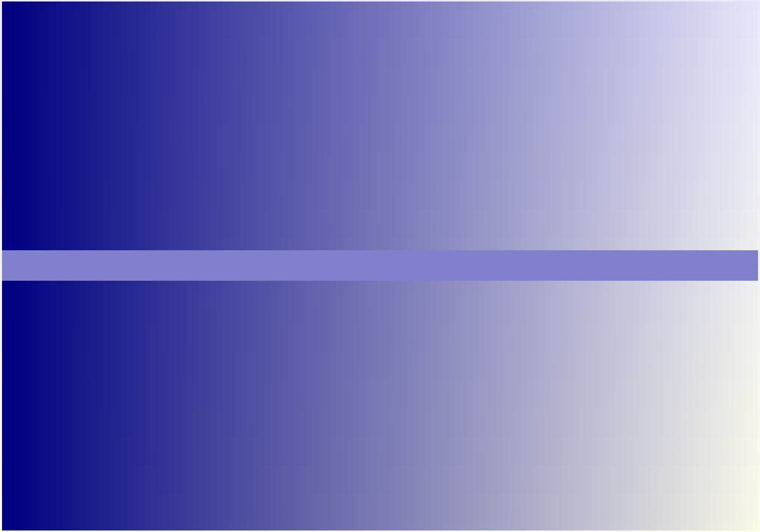
Red Bar



Green Bar

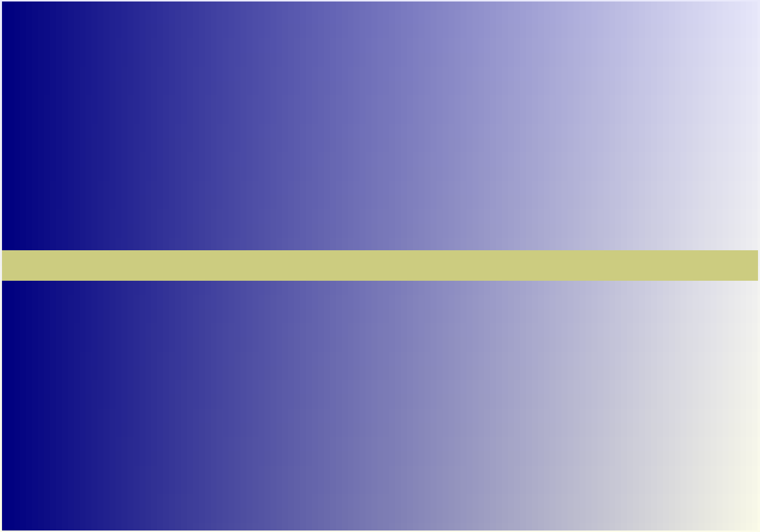


Blue Bar

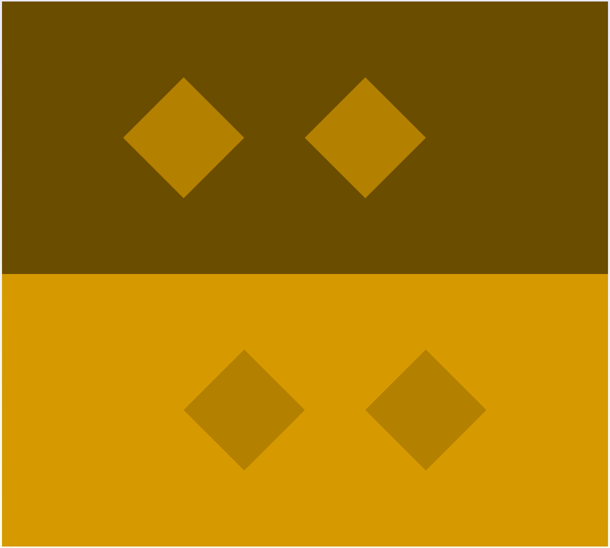


Yellow Bar

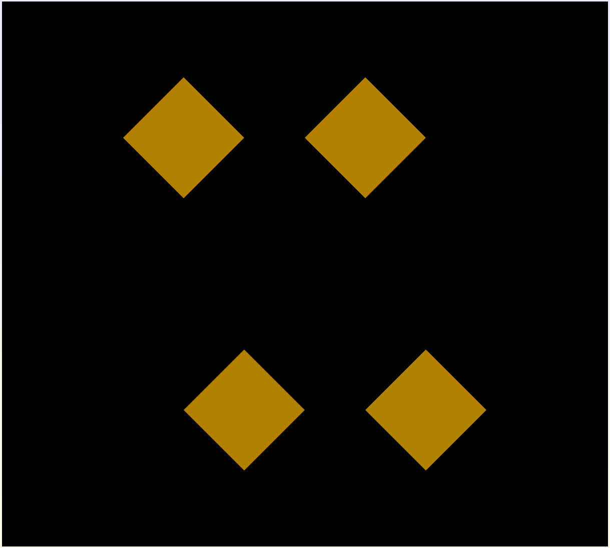
Yellow Bar



The Diamonds



The Diamonds

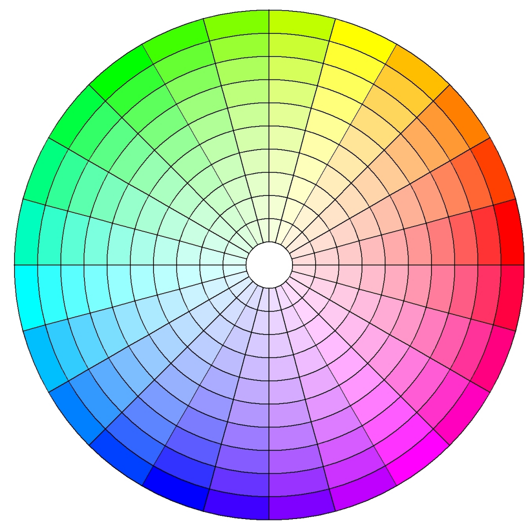


Luma and Gray Scale

- Luma approximates perceptual luminance
- Rec. 709 color video, BT. 709 digital color convex combinations
  - $Y = 0.2126R + 0.7152G + 0.0722B$
- Rec. 601 digital color convex combinations
  - $Y = 0.299R + 0.587G + 0.114B$
- Monitors are color balanced in a way such that $(R, G, B) = (g, g, g)$, for some $g \in [0, 1]$ has the appearance of gray
- The Luma difference between $(R, G, B) = (g, g, g)$ and $(R, G, B) = (g + \delta, g + \delta, g + \delta)$ is δ

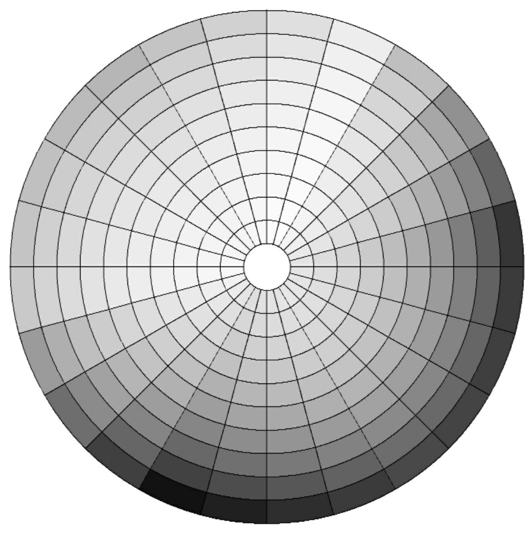
See http://en.wikipedia.org/wiki/YUV

HSB Slice 0



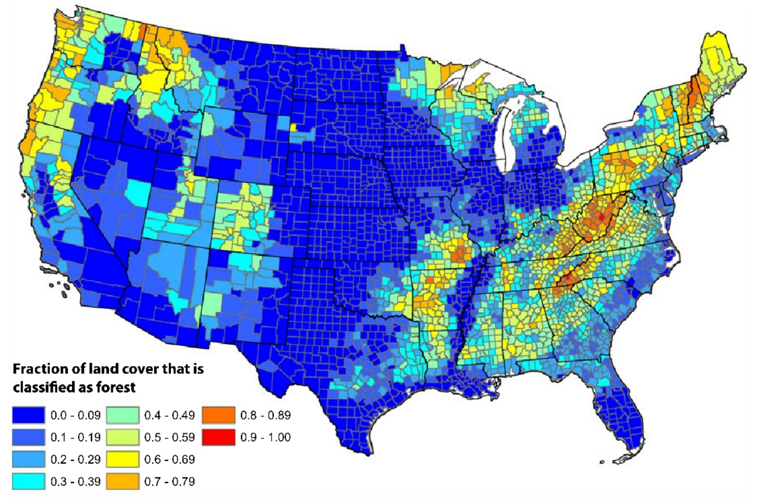
HSB Brightness=1; Outer Ring Saturation =1 and decreases 20% each ring inward.

HSB Gray



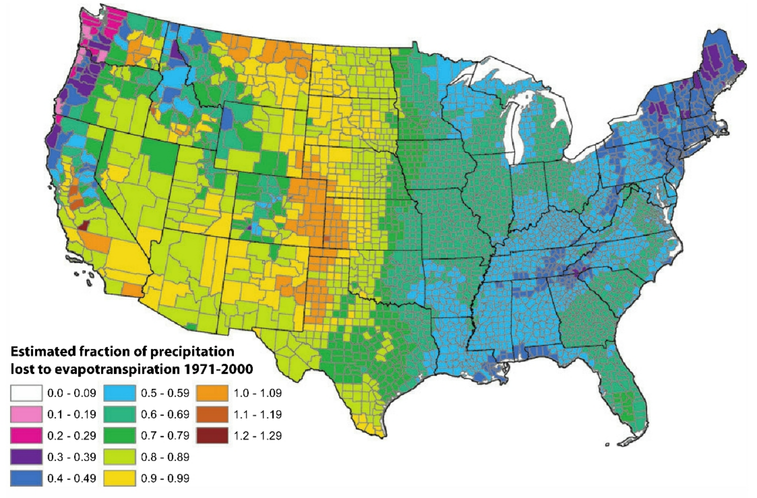
HSB Slice 0 Color converted to gray based on Luma

Potential Problems For Data Displayed in Color



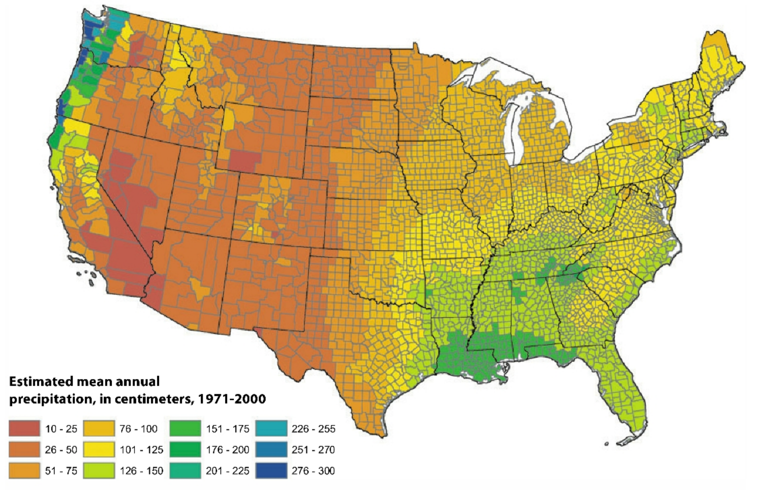
Notice that the fraction of land cover that is forrest that is in the interval 0.0-0.09 and that in the interval of 0.1-0.19 are in colors with related hue but very different luminances. There is an apparent perceptual division that is not a division that the data has. Sanford and Selnick 2013

Potential Problems For Data Displayed in Color



Notice that the precipitation fraction lost to evapotranspiration in that is in the interval .7-.79 and that in the interval of .8-.89 are in colors with related hue but very different luminances. This makes an apparent perceptual division that is not a division that the data has. Sanford and Selnick 2013

Potential Problems For Data Displayed in Color



Notice that in this map successive intervals of estimated mean annual precipitation have related colors and related luminances. The luminance difference for the interval 10-25 and 26-50 is too large.

Sanford and Selnick 2013

Equal Luma Colors

$$Y = \alpha R + \beta G + \gamma B$$

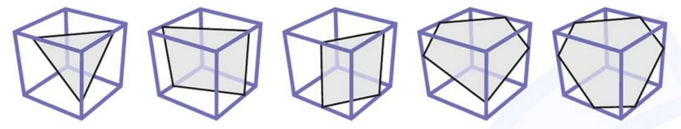
$$\begin{aligned} \alpha &= 0.2126 \\ \beta &= 0.7152 \\ \gamma &= 0.0722 \\ k &\in [0, 1] \end{aligned}$$

The set $E(k)$ of all colors that have Luma k is given by

$$E(k) = \left\{ \begin{pmatrix} R \\ G \\ B \end{pmatrix} \in [0, 1]^3 \;\middle|\; G = \frac{k - \alpha R - \gamma B}{\beta} \right\}$$

The points in $E(k)$ are the intersection of a plane with a cube.

Plane Cutting Cube



Different ways a plane can intersect a cube

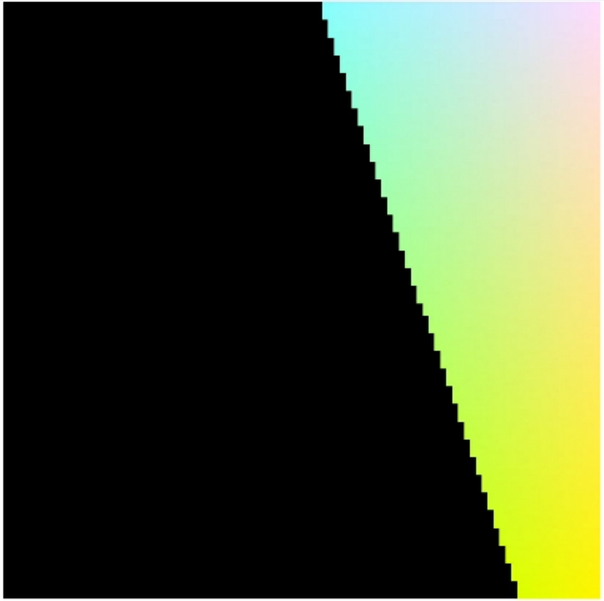
Equal Luma Series

The set $E(k)$ of all colors that have Luma k is given by

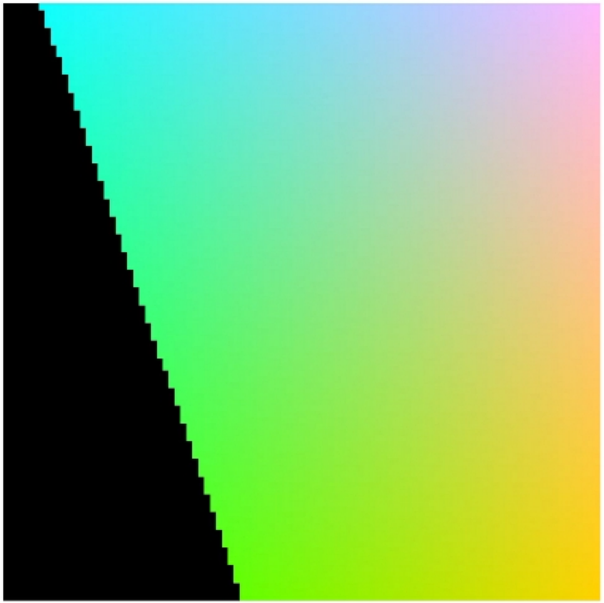
$$E(k) = \left\{ \begin{pmatrix} R \\ G \\ B \end{pmatrix} \in [0,1]^3 \;\middle|\; G = \frac{k - \alpha R - \gamma B}{\beta} \right\}$$

The points in $E(k)$ are the intersection of a plane with a cube. The following slides show different planes as Luma k is varied.

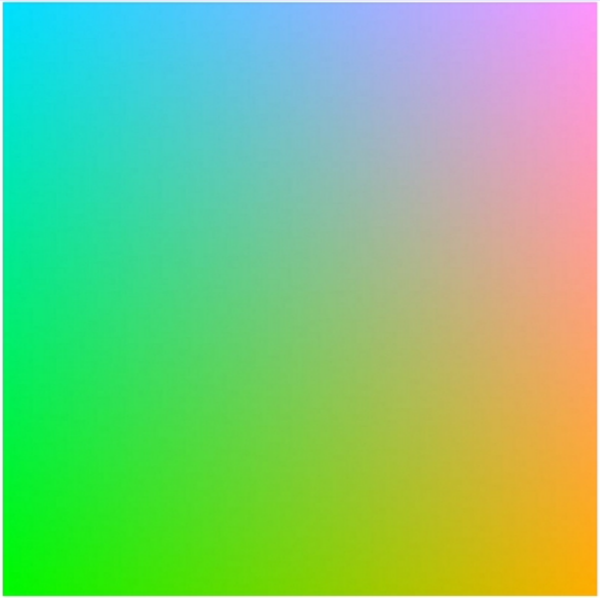
Equal Luma Colors: Luma = .9



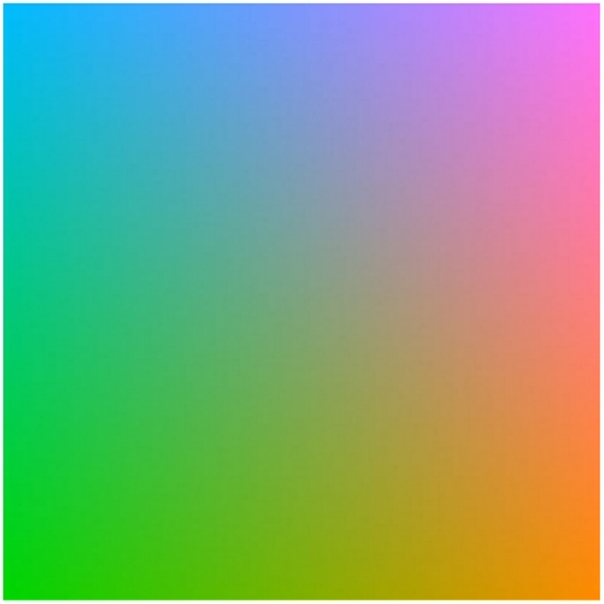
Equal Luma Colors: Luma = .8



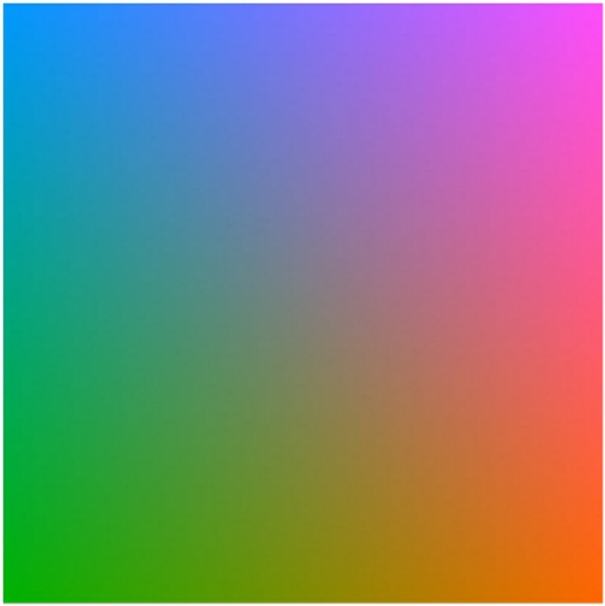
Equal Luma Colors: Luma = .7



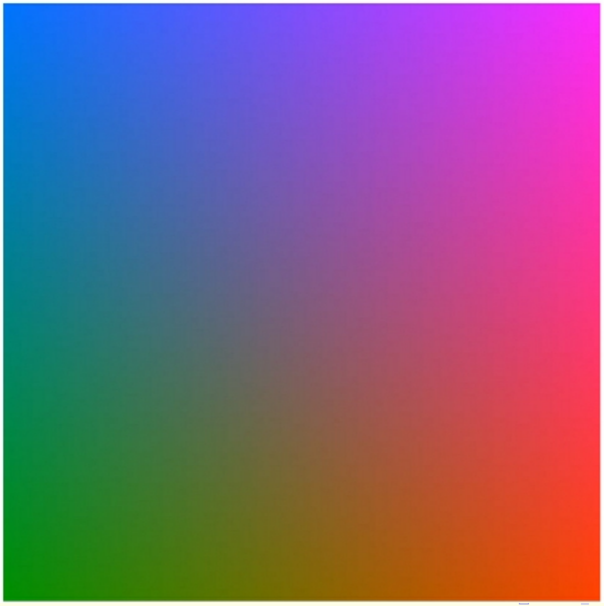
Equal Luma Colors: Luma = .6



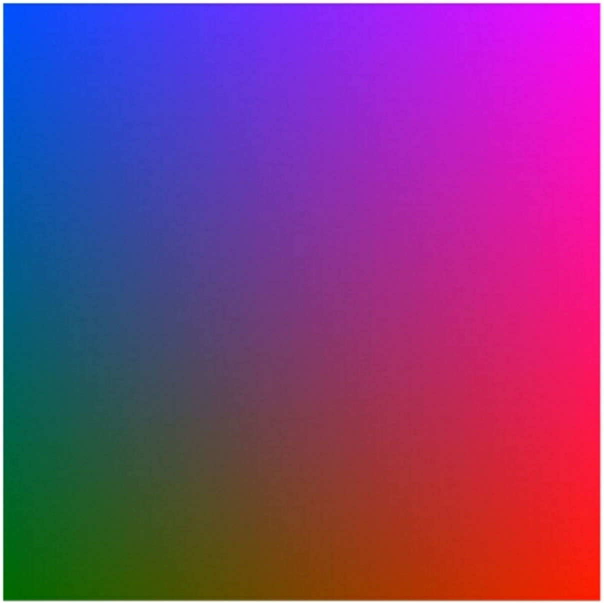
Equal Luma Colors: Luma = .5



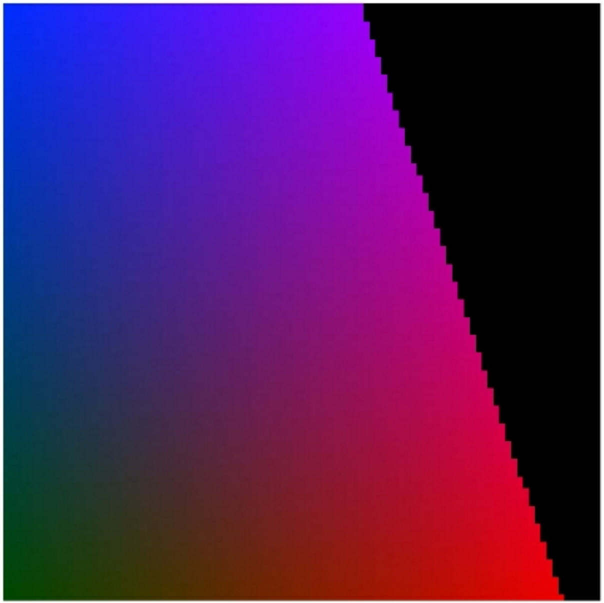
Equal Luma Colors: Luma = .4



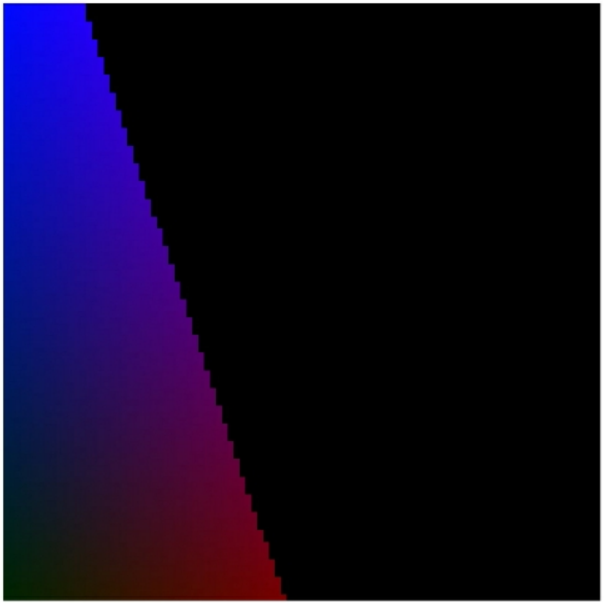
Equal Luma Colors: Luma = .3



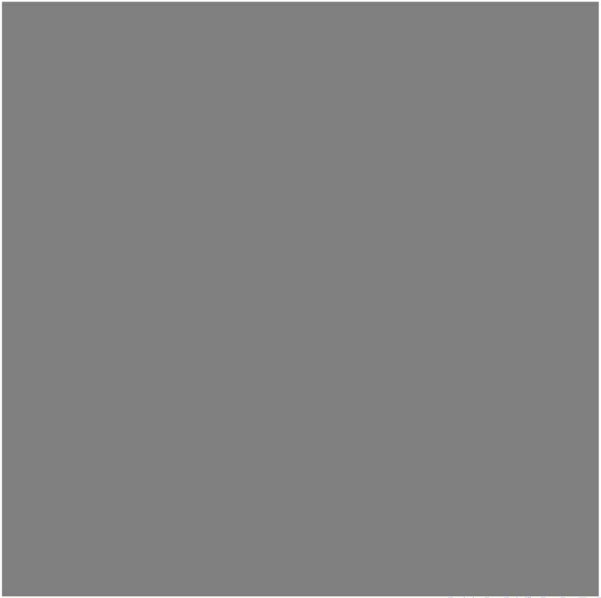
Equal Luma Colors: Luma = .2



Equal Luma Colors: Luma = .1



Equal Luma Colors Changed to Gray Scale: Luma = .5



Inverse Gamma

The inverse gamma transformation made by digital cameras is given by

$$
\begin{aligned}
R' &= \begin{cases} 12.92R & \text{if } R < .00304 \\ 1.055R^{.416667} - .055 & \text{otherwise} \end{cases} \\
G' &= \begin{cases} 12.92G & \text{if } G < .00304 \\ 1.055G^{.416667} - .055 & \text{otherwise} \end{cases} \\
B' &= \begin{cases} 12.92B & \text{if } B < .00304 \\ 1.055B^{.416667} - .055 & \text{otherwise} \end{cases} \\
L &= .2126R + .7152G + .0722B
\end{aligned}
$$

The inverse gamma transformation allows more bits to be allocated to the low luminance areas.
See http://en.wikipedia.org/wiki/SRGB

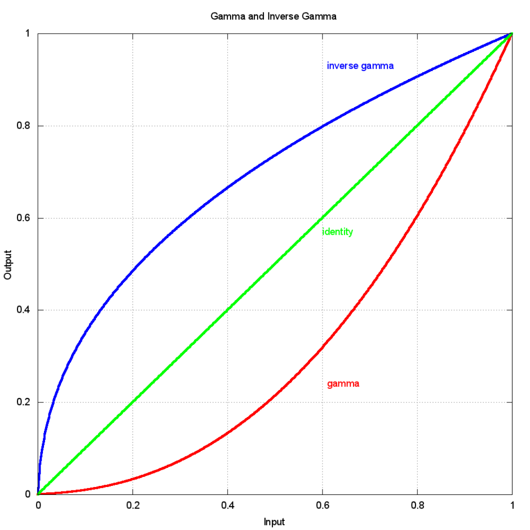
Gamma

The gamma transformation made by computers undoes the transformation made by the digital cameras and is given by

$$
\begin{aligned}
R' &= \begin{cases} R/12.92 & \text{if } R < .03928 \\ \left(\frac{R+.055}{1.055}\right)^{2.4} & \text{otherwise} \end{cases} \\
G' &= \begin{cases} G/12.92 & \text{if } G < .03928 \\ \left(\frac{G+.055}{1.055}\right)^{2.4} & \text{otherwise} \end{cases} \\
B' &= \begin{cases} B/12.92 & \text{if } B < .03928 \\ \left(\frac{B+.055}{1.055}\right)^{2.4} & \text{otherwise} \end{cases} \\
L &= .2126R' + .7152G' + .0722B'
\end{aligned}
$$

See http://en.wikipedia.org/wiki/SRGB

Gamma and Inverse Gamma



Composition of gamma with inverse gamma or vica-versa is the identify function.

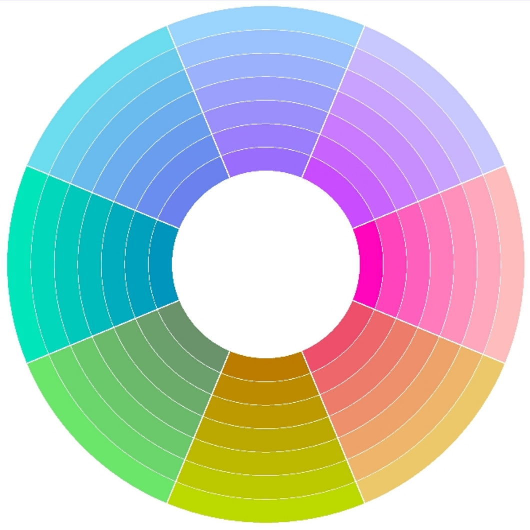
Luminance

Colors set by a luminance computed mathematically, must be preprocessed by an inverse gamma correction to offset the gamma correction that the computer is going to make.

$$L = .2126R + .7152G + .0722B$$

The three coefficients form the middle row of the RGB-to-XYZ color transformation matrix associated with the D65 Illuminant. This is defined on later slides.

Equal Luminance Color Wheel



Equal Luminance in rings; Outside ring has luminance .82.
Luminance decreases by 16% as rings move to center

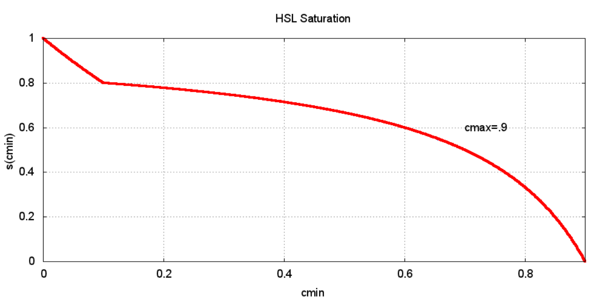
Hue Saturation Lightness: HSL

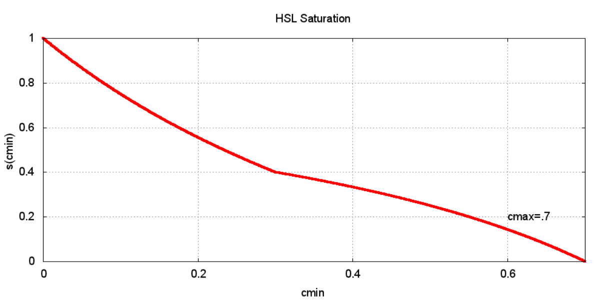
- Hue is same is HSB
- $C_{max} = \max\{R, G, B\}$
- $C_{min} = \min\{R, G, B\}$
- $\Delta = C_{max} - C_{min}$
- Lightness: $L = \frac{C_{max}+C_{min}}{2}$
- Saturation: Differently defined than in HSB
- Saturation $S = \begin{cases} 0 & \Delta = 0 \\ \frac{\Delta}{2\min\{L, 1-L\}} & \text{Otherwise} \end{cases}$
- Saturation $S = \begin{cases} 0 & \Delta = 0 \\ \frac{\Delta}{2L} & \Delta > 0 \text{ and } L \leq \frac{1}{2} \\ \frac{\Delta}{2(1-L)} & \Delta > 0 \text{ and } L > \frac{1}{2} \end{cases}$

HSL Saturation

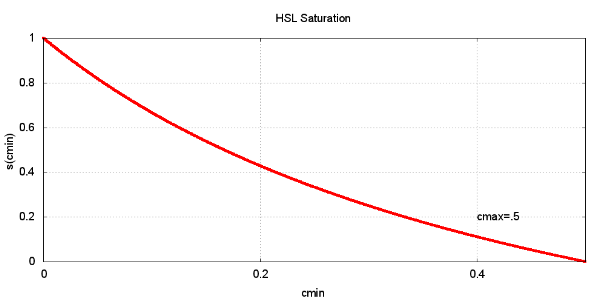
$$
\begin{aligned}
C_{max} &= \max\{R, G, B\} \\
C_{min} &= \min\{R, G, B\} \\
S(C_{max}, C_{min}) &= \frac{C_{max} - C_{min}}{1 - |C_{max} + C_{min} - 1|} \\
&= \begin{cases} \frac{C_{max} - C_{min}}{2 - (C_{max} + C_{min})} & 1 < C_{max} + C_{min} < 2 \\ \frac{C_{max} - C_{min}}{C_{max} + C_{min}} & 0 < C_{max} + C_{min} \leq 1 \end{cases}
\end{aligned}
$$

HSL Saturation





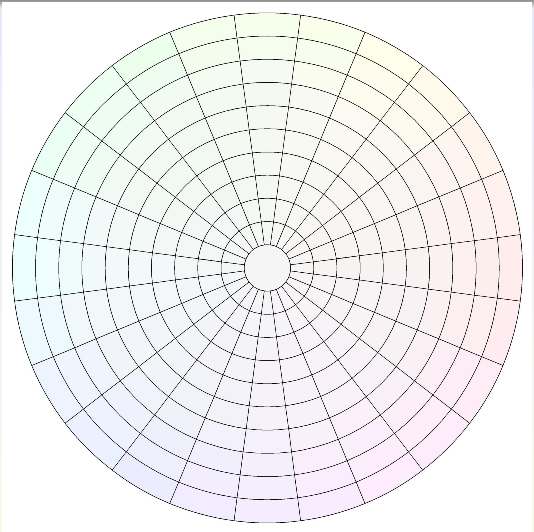
HSL Saturation



Understanding HSL Saturation

- C_{min} represents the amount of white that is added to the color
- When $C_{min} = 0$, $C_{max} > 0$, $S = 1$
- When $C_{min} = C_{max}$, $S = 0$
- As C_{min} increases from 0 to C_{max}, S decreases from 1 to 0
- $L < .5$
  - $C_{max} = L(1 + S)$
  - $C_{min} = L(1 - S)$
- $L > .5$
  - $C_{max} = L + S(1 - L) = L(1 - S) + S$
  - $C_{min} = L - S(1 - L) = L(1 + S) - S$

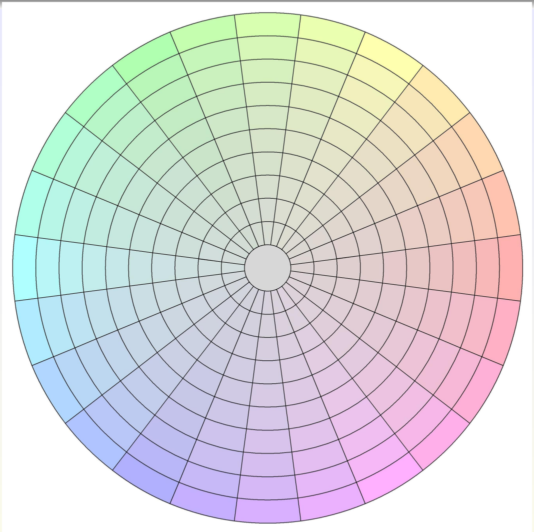
HSL Saturation



$$L = .9627 = .5(1.14)^5$$

$S = 1$ on outer ring and decreases 20% each successive ring toward center.

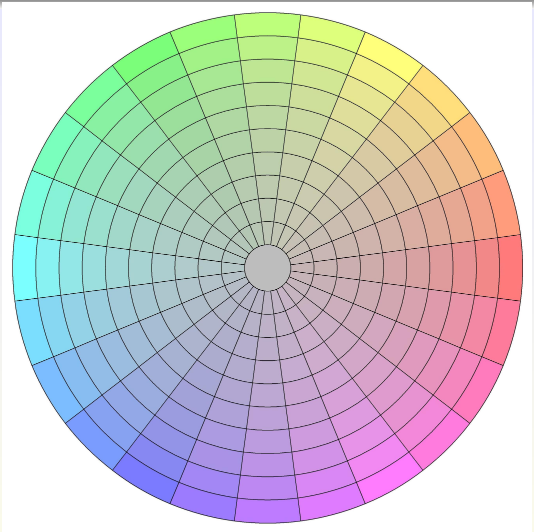
HSL Saturation



$$L = .8445 = .5(1.14)^4$$

$S = 1$ on outer ring and decreases 20% each successive ring toward center.

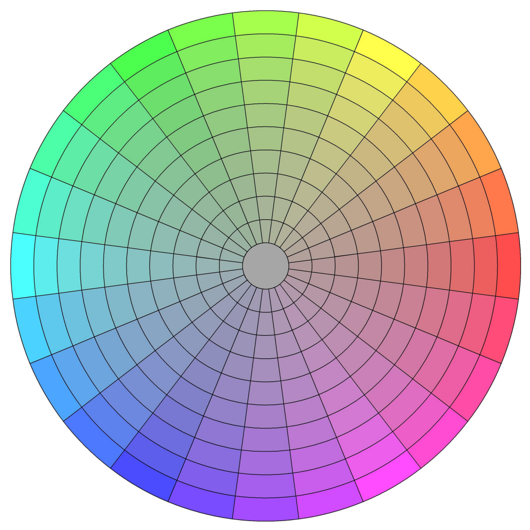
HSL Saturation



$$L = .7408 = .5(1.14)^3$$

$S = 1$ on outer ring and decreases 20% each successive ring toward center.

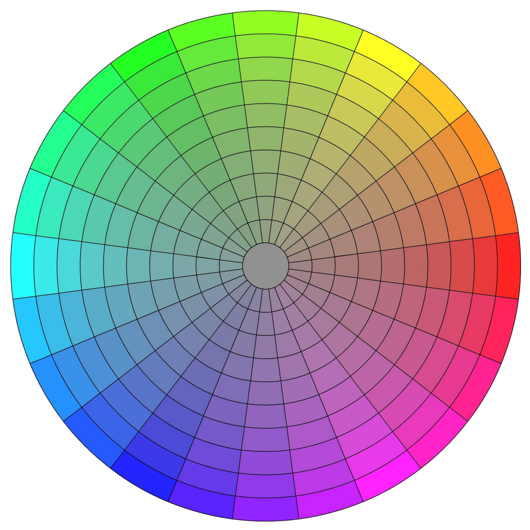
HSL Saturation



$$L = .6498 = .5(1.14)^2$$

$S = 1$ on outer ring and decreases 20% each successive ring toward center.

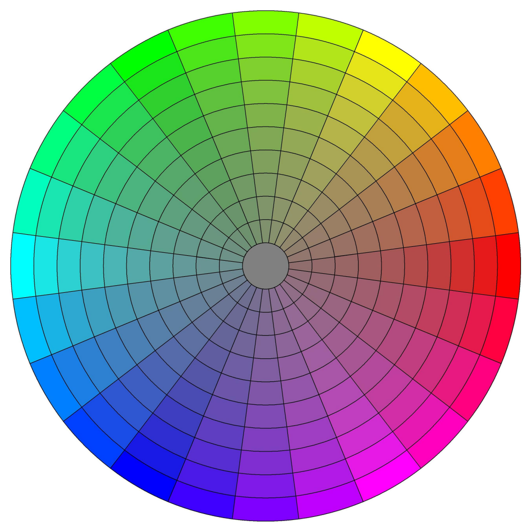
HSL Saturation



$$L = .57 = .5(1.14)^1$$

$S = 1$ on outer ring and decreases 20% each successive ring toward center.

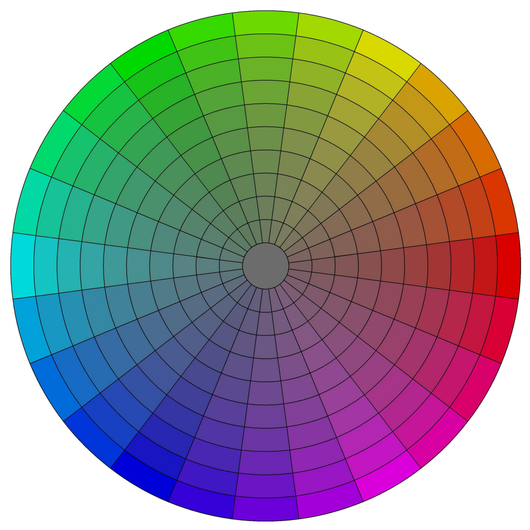
HSL Saturation



$$L = .5 = .5(1.14)^0$$

$S = 1$ on outer ring and decreases 20% each successive ring toward center.

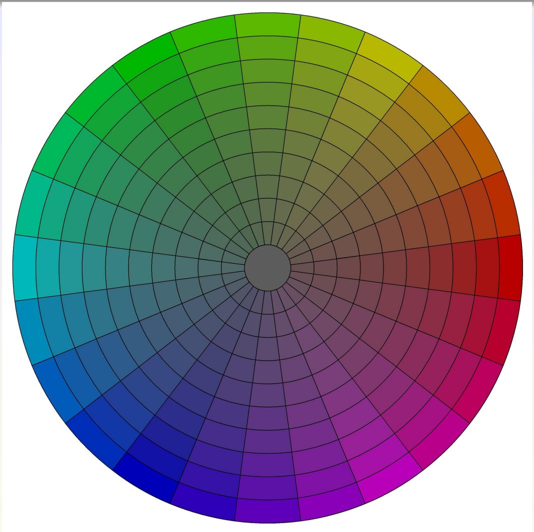
HSL Saturation



$$L = .4 = .5(.8)^1$$

$S = 1$ on outer ring and decreases 20% each successive ring toward center.

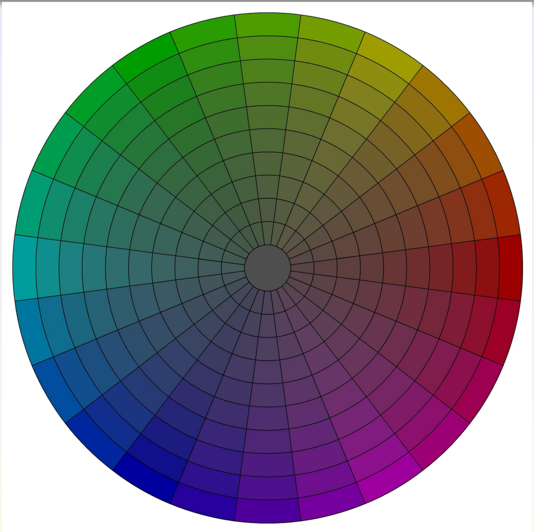
HSL Saturation



$$L = .32 = .5(.8)^2$$

$S = 1$ on outer ring and decreases 20% each successive ring toward center.

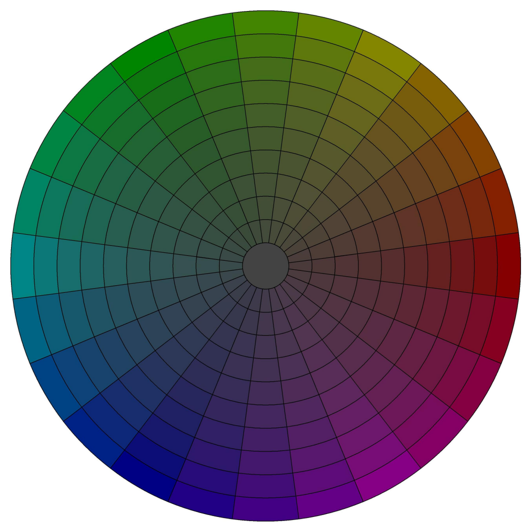
HSL Saturation



$$L = .256 = .5(.8)^3$$

$S = 1$ on outer ring and decreases 20% each successive ring toward center.

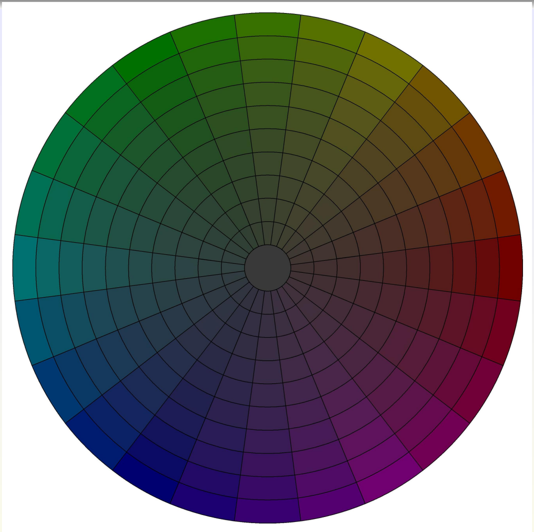
HSL Saturation



$$L = .2048 = .5(.8)^4$$

$S = 1$ on outer ring and decreases 20% each successive ring toward center.

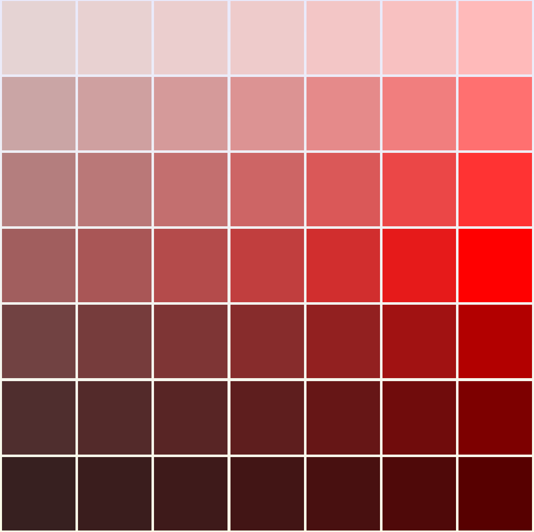
HSL Saturation



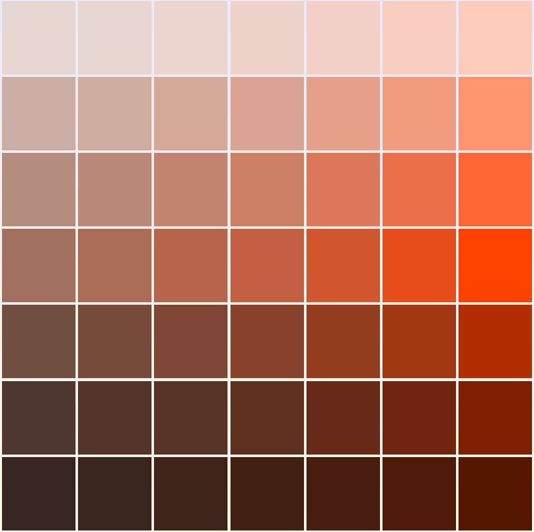
$$L = .16384 = .5(.8)^5$$

$S = 1$ on outer ring and decreases 20% each successive ring toward center.

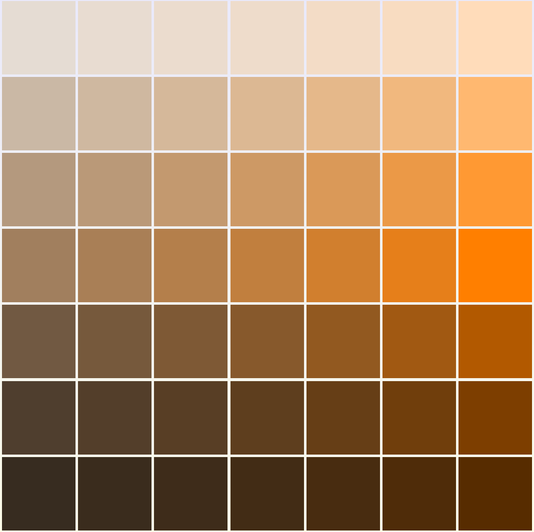
HSL Slab 0°



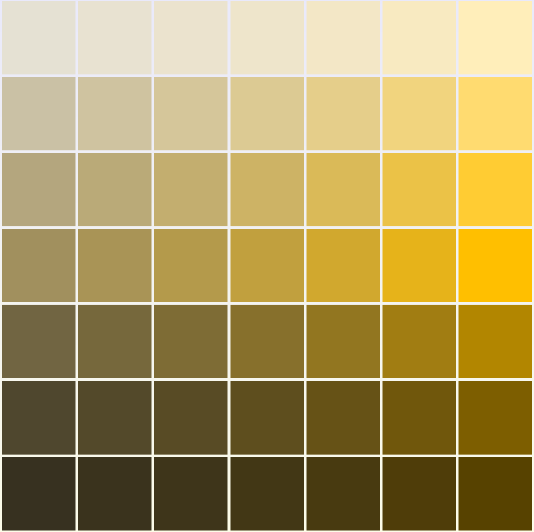
HSL Slab 15°



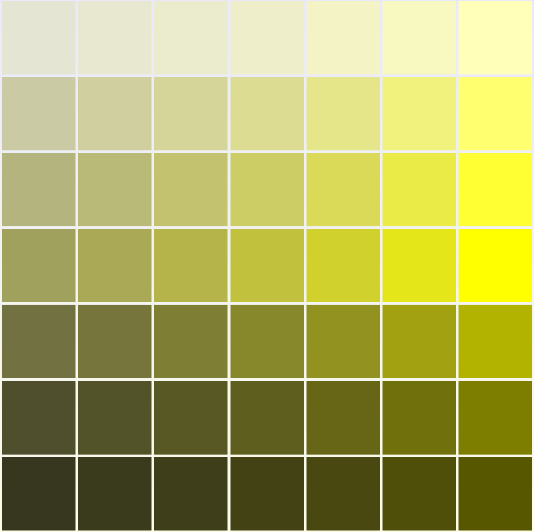
HSL Slab 30°



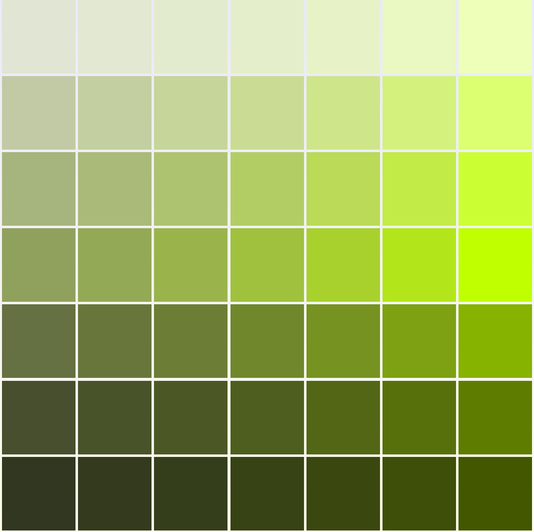
HSL Slab 45°



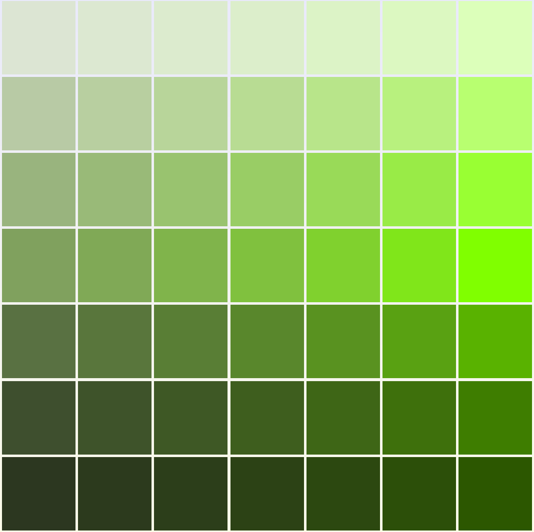
HSL Slab 60°



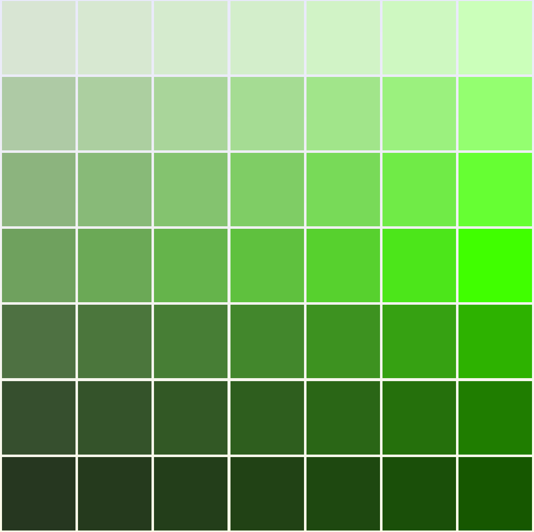
HSL Slab 75°



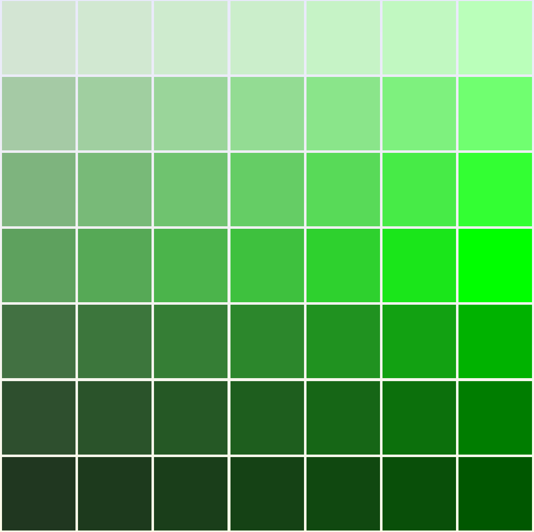
HSL Slab 90°



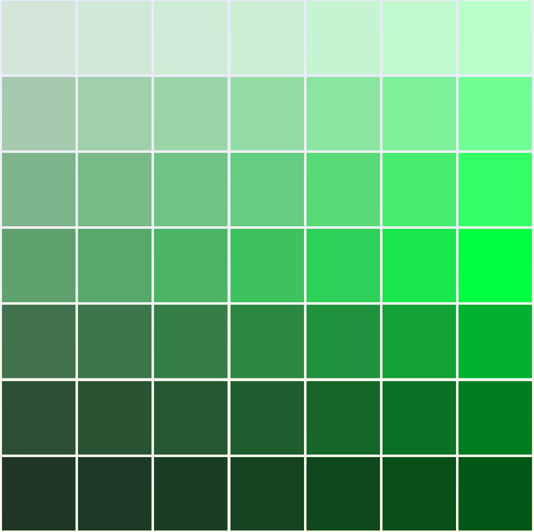
HSL Slab 105°



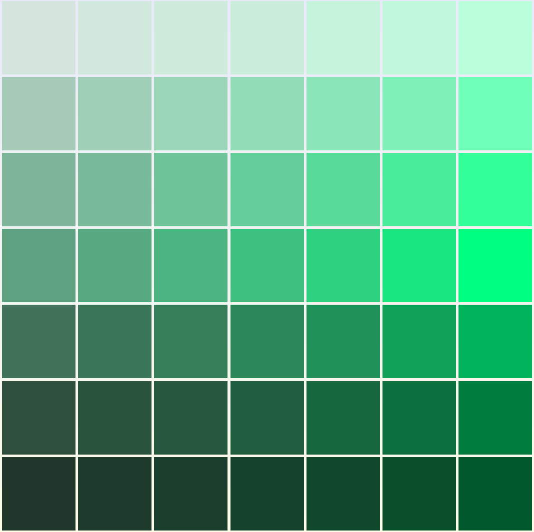
HSL Slab 120°



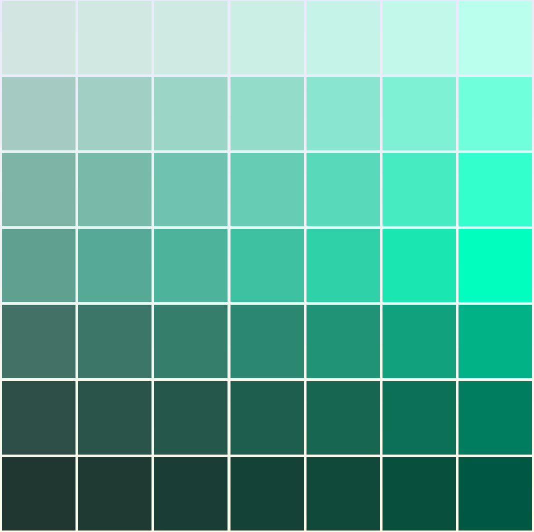
HSL Slab 135°



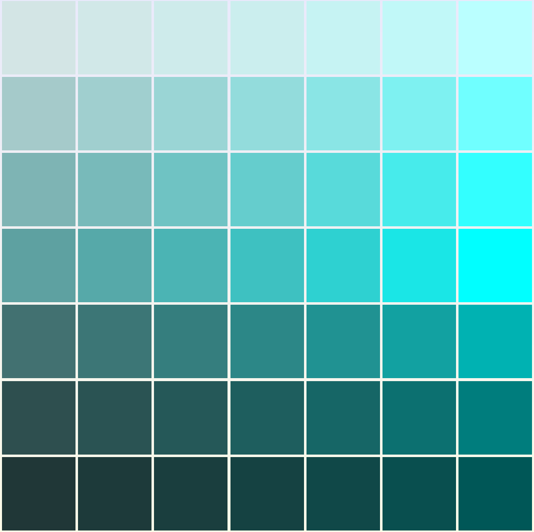
HSL Slab 150°



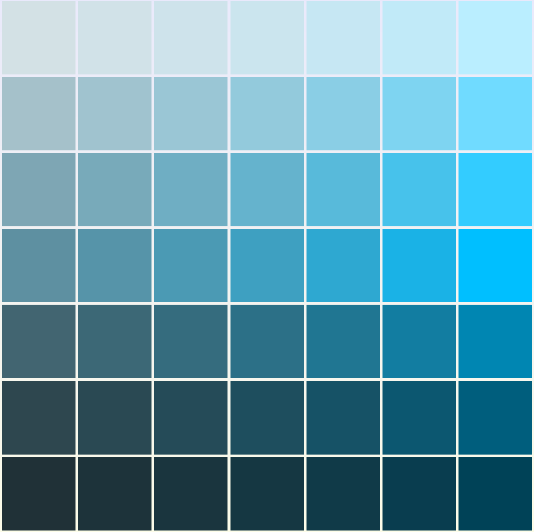
HSL Slab 165°



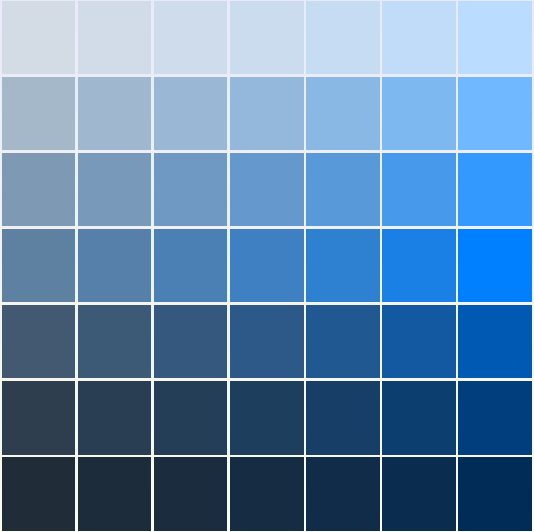
HSL Slab 180°



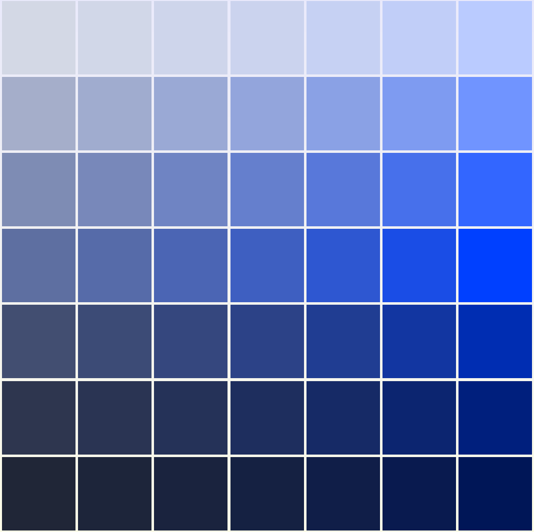
HSL Slab 195°



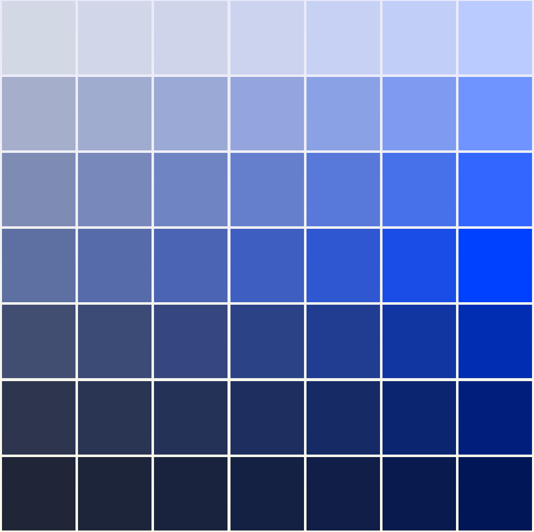
HSL Slab 210°



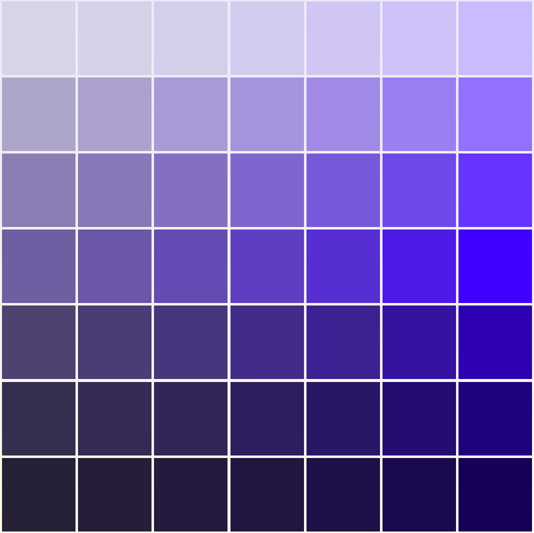
HSL Slab 225°



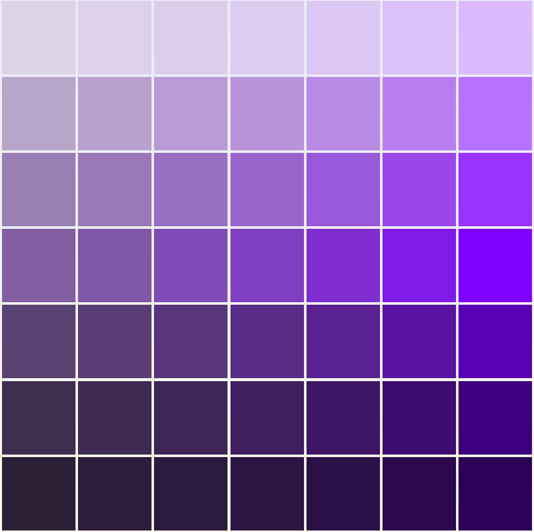
HSL Slab 240°



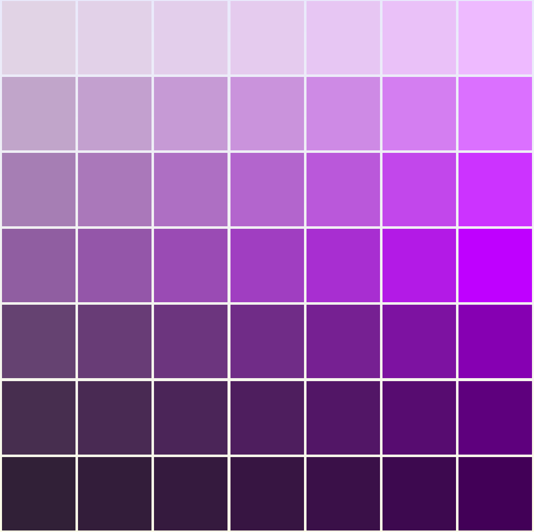
HSL Slab 255°



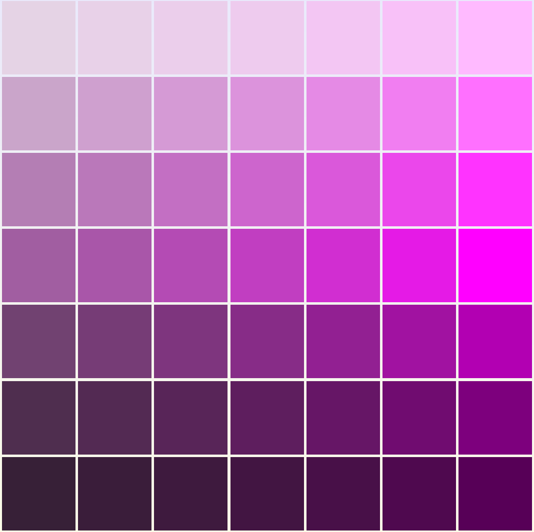
HSL Slab 270°



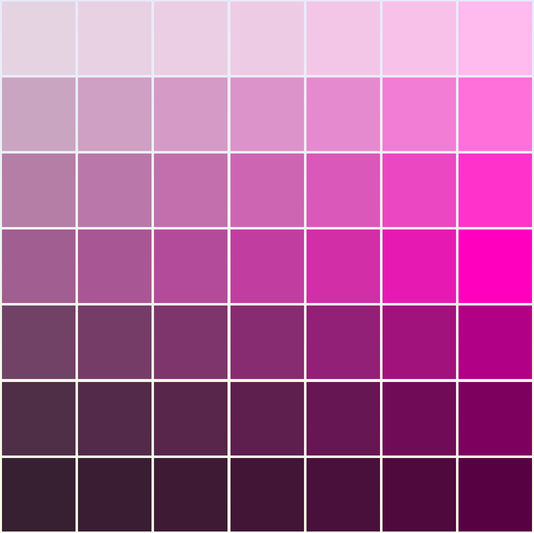
HSL Slab 285°



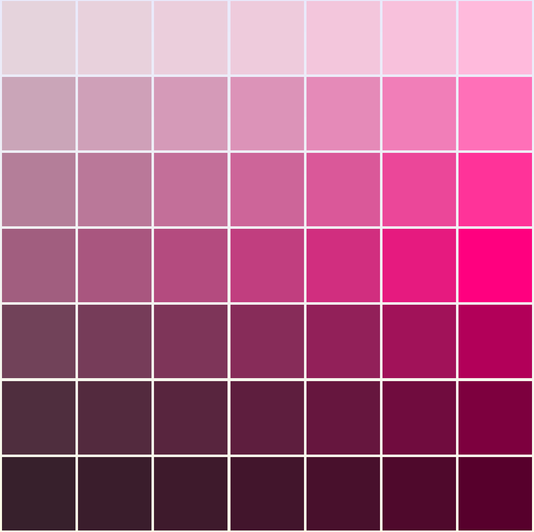
HSL Slab 300°



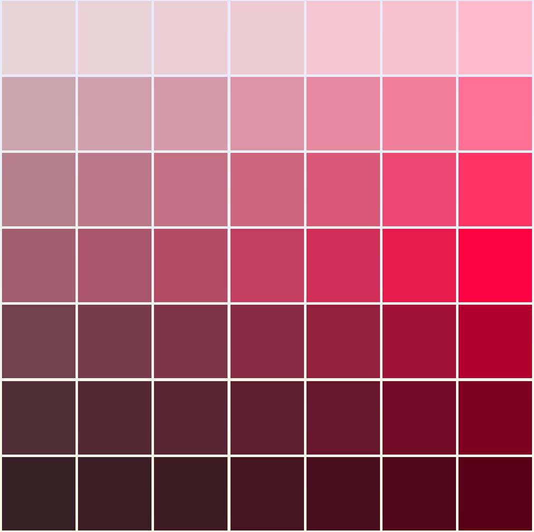
HSL Slab 315°



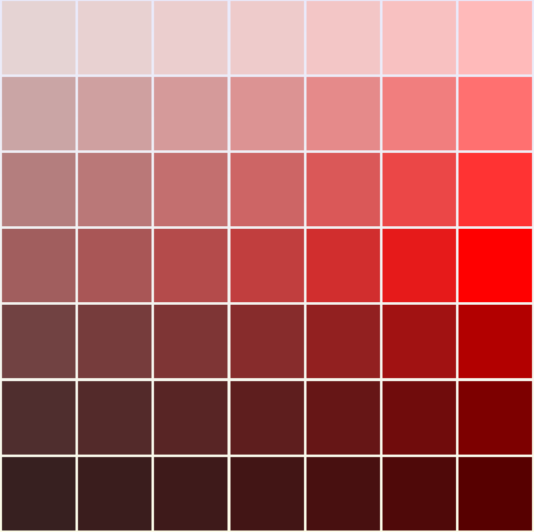
HSL Slab 330°



HSL Slab 345°

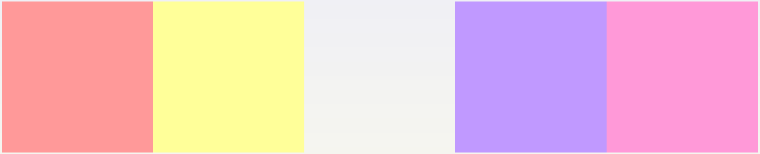


HSL Slab 360°



HSL

- HSL is not perceptually uniform when saturation and lightness are held constant
  - The perceived difference between colors is not proportional to their separation angle in hue



0.0 60.0 263.0 323.0

The saturation is 1 and the lightness is .8 for each color. The difference in hue between each pair is 60°. One pair is perceptually more similar than the other pair.

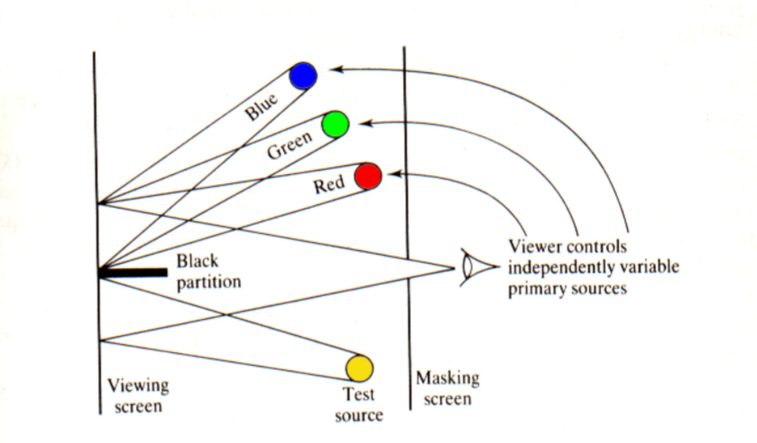
Wright and Guild

- W.D. Wright at Imperial College in London
- J. Guild at National Physical Laboratory
- Worked in the late 1920's
- Lead to the CIE 1931 standard colorimetric observer

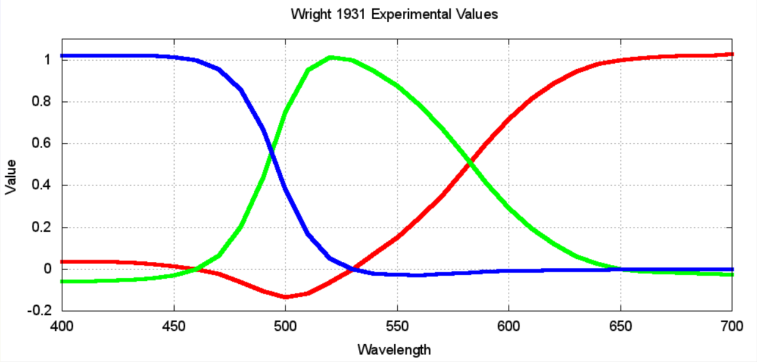
Wright's Measurements

- Ten normal color vision non-fatigued observers
- Square visual field subtended a 2° to insure light falls on foveal region
- Level of Illumination is high enough to ensure exclusive cone vision
- Horizontally divided into two equal rectangles
  - The first illuminated by a monochromatic spectral light
  - The second a variable brightness mixture of red $(650nm)$, green $(530nm)$, blue $(460nm)$
  - The mixture could be independently varied by the observer to match the monochromatic spectral source

Wright's Experimental Setup



Wright's Original Experimental Values



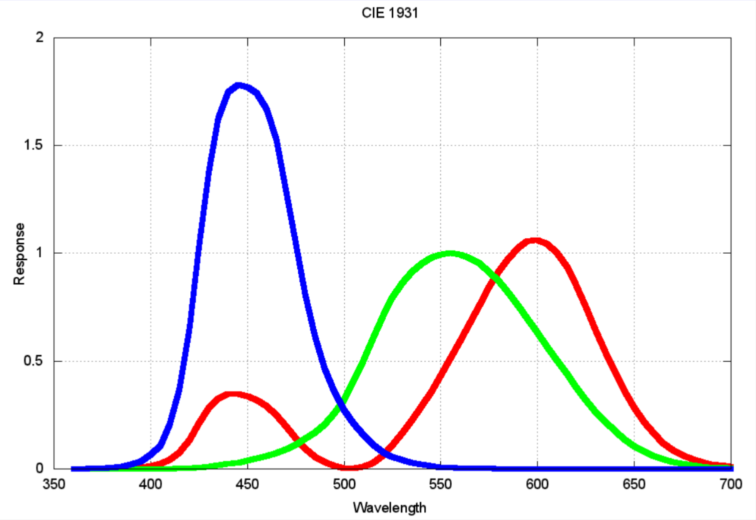
Negative values mean that this amount had to be added to the test light source to make the match.

CIE XYZ Color Space

- Commission Internationale de l'Èclairage
- 1931
- *Standard Observer* 2° *and* 10°
- Color Matching Functions $\bar{x}, \bar{y}, \bar{z}$
- Producing Non-negative Tristimulus Values X, Y, Z
- Every perceived color is associated with a non-negative mixture of X, Y, Z

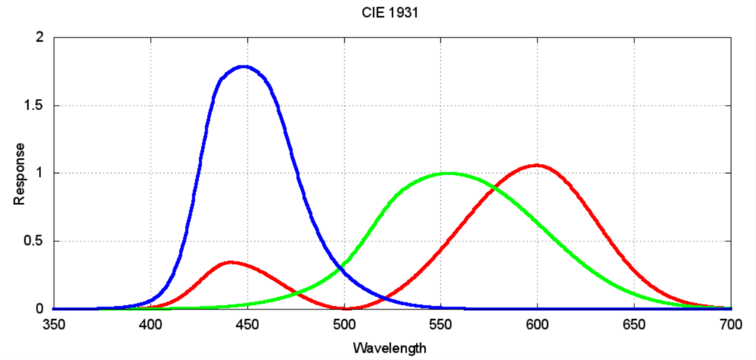
$$
\begin{aligned}
X &= \int e(\lambda)\bar{x}(\lambda)d\lambda \\
Y &= \int e(\lambda)\bar{y}(\lambda)d\lambda \\
Z &= \int e(\lambda)\bar{z}(\lambda)d\lambda
\end{aligned}
$$

CIE 1931 Color Matching Functions



Tabulated Data From www.cvrl.org

CIE Color Matching Functions



$\bar{x}$ $\bar{y}$ $\bar{z}$

Chris Wyman, Peter-Pike Sloan, Peter Shirley, *Simple Analytic Approximations to the CIE XYZ Color Matching Functions* **Journal of Computer Graphics Techniques**, Vol 2, No. 2, 2013, pp 1-11.

RGB To XYZ With Illuminant E

E is a designator of equal energy radiance in the visible band

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} .4887 & .3107 & .2006 \\ .1762 & .8130 & .0108 \\ .0000 & 0.0102 & 0.9898 \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = \begin{pmatrix} 2.3706 & -.9000 & -.4706 \\ -0.5139 & 1.4253 & .0886 \\ 0.0053 & -.01469 & 1.0094 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

R $<$ 0 or G $<$ 0 or B $<$ 0 do not correspond to any perceived color.

sRGB To XYZ With Illuminant D65

D65 is a designator for a simulated daylight

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} .4125 & .3576 & .1804 \\ .2127 & .7152 & .0722 \\ .0193 & 0.1192 & 0.9503 \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = \begin{pmatrix} 3.2404 & -1.5371 & -.4985 \\ -0.9693 & 1.8760 & .0416 \\ 0.0556 & -.2040 & 1.0572 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

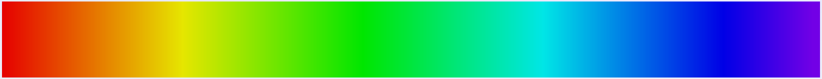
(X,Y,Z) values that are associated with R < 0 or G < 0 or B < 0 do not correspond to any perceived color. Similarly for R > 1 or G > 1 or B > 1

Tristimulus Values

$$
\begin{aligned}
x &= \frac{X}{X+Y+Z} \\
y &= \frac{Y}{X+Y+Z} \\
z &= \frac{Z}{X+Y+Z} \\
x+y+z &= 1 \\
z &= 1-x-y
\end{aligned}
$$

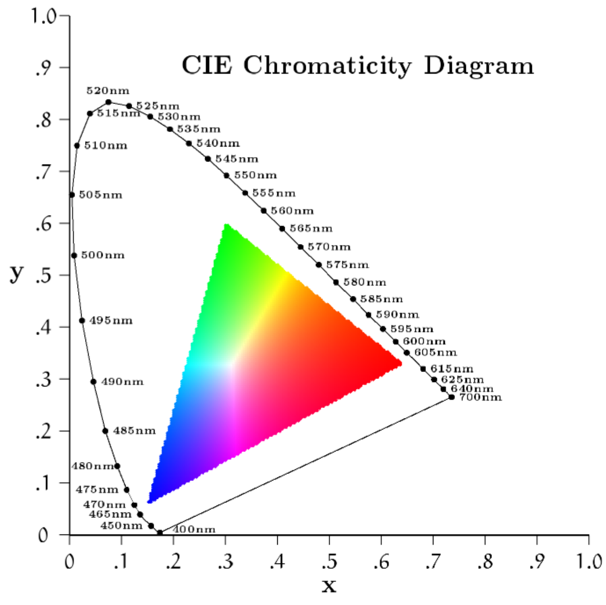
- All colors have unique tristimulus coordinates (x, y, z)
- Some (x, y, z) do not correspond to any perceived color

Tristimulus Values



- Each spectral color has tristimulus coordinates (x, y, z)
- Any perceived color is a mixture of spectral colors
- The convex hull of the tristimulus coordinates of spectral colors is the set of tristimulus values that correspond to perceived colors
- Every color has unique tristimulus coordinates (x, y, z)
- Some (x, y, z) do not correspond to any perceived color
- The set of colors that can be displayed by a mixture of Red, Green, Blue is a proper subset of all the colors that can be perceived

1931 CIE Chromaticity Diagram



1948 Richard Hunter

- Built a photoelectric color-difference meter
- Based on CIE XYZ
- 3 values to quantify the color: L, a, b
- Lightness L-axis
- Hering's red-green opponent colors a-axis
- Hering's blue-yellow opponent colors b-axis
- Used square root to approximate the perceptual response

Hunter, R. S., Photoelectric Color-Difference Meter, J. Opt. Soc. Am. 38, 661 (1948).

1958 Glasser et. al.

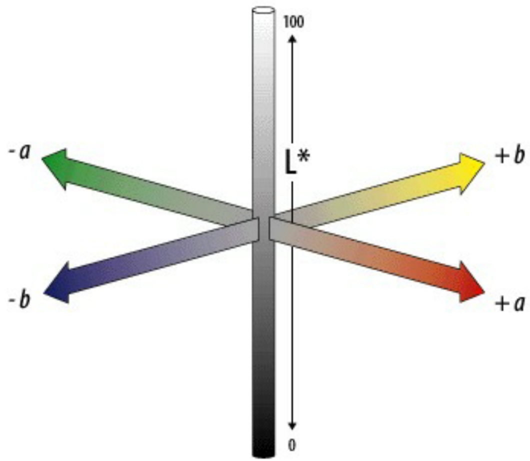
- Visually uniform color coordinate system
- Cube root approximates the perceptual response as a function of physical energy
- Lightness L-axis
- Hunter's red-green opponent colors a*-axis
- Hunters's blue-yellow opponent colors b*-axis

Glasser, L. G., McKinney, A. H., Reilly, C. D., and Schnelle, P. D., Cube-Root Color Coordinate System, J. Opt. Soc. Am. 48, 736 (1958).

CIE-La*b* 1976

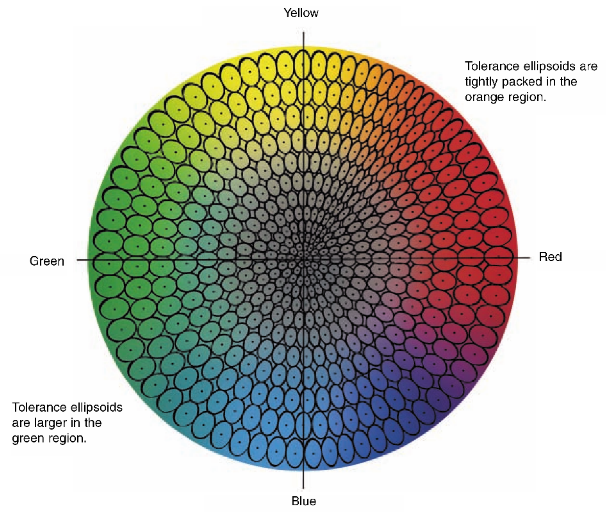
- Uniform: distance is proportional to discriminability
- Tristimulus XYZ
- Reference White X_n, Y_n, Z_n
- L Luminance
  - $L = 100$ represents the adaptation white
  - $L = 0$ represents the ideal black
- C Chrominance
- a^*, b^* opponent colors
  - a^*-axis red-green
  - b^*-axis blue-yellow
- Euclidean Distance in La*b* space is related to perceptual difference

CIE-La*b* 1976



Chrominance: $C^* = (a^{*2} + b^{*2})^{1/2}$
(Misleading Diagram)

CIE-La*b* 1976 Tolerance Ellipsoids



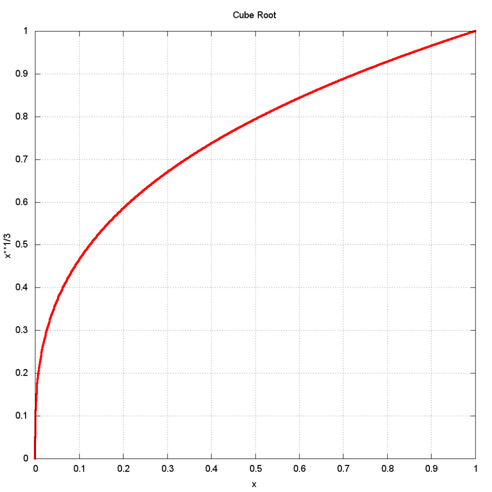
The ellipsoids in the orange area of color space are longer and narrower than the broader and rounder ones in the green area. The size and shape of the ellipsoids also change as the color varies in chroma and/or lightness.

A Guide To Understanding Color Communication, X-Rite Inc.

CIE-La*b*

$$
\begin{aligned}
L &= 116 f\left(\frac{Y}{Y_n}\right) - 16 \\
a^* &= 500\left[f\left(\frac{X}{X_n}\right) - f\left(\frac{Y}{Y_n}\right)\right] \\
b^* &= 200\left[f\left(\frac{Y}{Y_n}\right) - f\left(\frac{Z}{Z_n}\right)\right] \\
C^* &= ({a^*}^2 + {b^*}^2)^{1/2} \\
h &= atan2(b^*, a^*) \\
f(x) &= \begin{cases} x^{1/3}, & x > \left(\frac{6}{29}\right)^3 \\ \frac{1}{3}\left(\frac{29}{6}\right)^2 x + \frac{4}{29}, & \text{otherwise} \end{cases}
\end{aligned}
$$

Cube Root



A change from 0 to .1 produces a change of 0 to about .47.
A change from .5 to 1 produces a change of about .8 to 1.

The Zero Point

When is $(a^*, b^*) = (0, 0)$?

$$\begin{aligned} f\left(\frac{X}{X_n}\right) - f\left(\frac{Y}{Y_n}\right) &= 0 \\ f\left(\frac{X}{X_n}\right) &= f\left(\frac{Y}{Y_n}\right) \\ \frac{X}{X_n} &= \frac{Y}{Y_n} \end{aligned}$$

Now, $\frac{X}{X_n} = \frac{Y}{Y_n}$ happens if and only if for some constant α

$$\begin{aligned} X &= \alpha X_n \\ Y &= \alpha Y_n \end{aligned}$$

The Zero Point

$$\begin{aligned} X &= \alpha X_n \\ Y &= \alpha Y_n \end{aligned}$$

When $\alpha = 1$

$$\begin{aligned} X &= X_n \\ Y &= Y_n \end{aligned}$$

When $\alpha = 0$

$$\begin{aligned} X &= 0 \\ Y &= 0 \end{aligned}$$

All Black and All White both map to the Zero Point in CIE La*b* space.

La*b* to XYZ

$$
\begin{aligned}
fy &= \frac{L+16}{116} \\
fx &= \frac{a^*}{500.} + fy \\
fz &= fy - \frac{b^*}{200} \\
Y &= Y_n \begin{cases} fy * fy * fy & fy > \frac{6}{29} \\ \frac{108}{841}\left(fy - \frac{4}{29}\right), & \text{otherwise} \end{cases} \\
X &= X_n \begin{cases} fx * fx * fx, & fx > \frac{6}{29} \\ \frac{108}{841}\left(fx - \frac{4}{29}\right), & \text{otherwise} \end{cases} \\
Z &= Z_n \begin{cases} fz * fz * fz, & fz > \frac{6}{29} \\ \frac{108}{841}\left(fz - \frac{4}{29}\right), & \text{otherwise} \end{cases}
\end{aligned}
$$

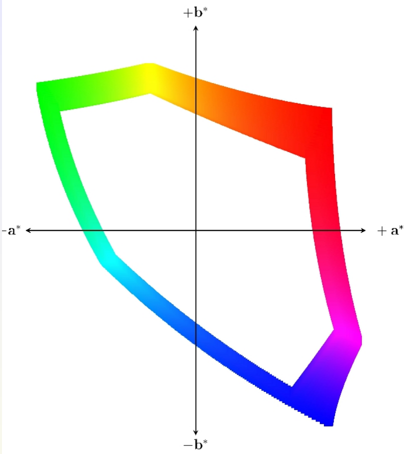
XYZ to RGB

D65 Simulated Daylight

$$\begin{pmatrix} R \\ G \\ B \end{pmatrix} = \begin{pmatrix} 3.2404542 & -1.5371385 & -.4985314 \\ -0.9692660 & 1.8760108 & .0415560 \\ 0.0556434 & -.2040259 & 1.0572252 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

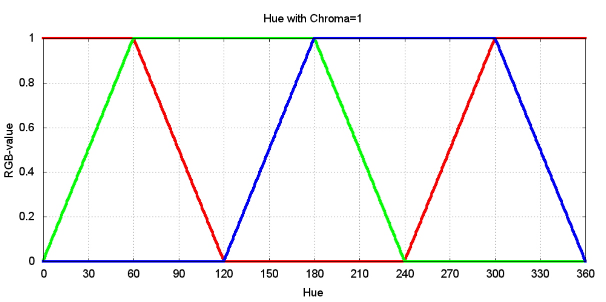
R <0 or G <0 or B <0 do not correspond to any perceived color. Similarly for R >1 or G >1 or B >1.

CIE La*b* Color Space



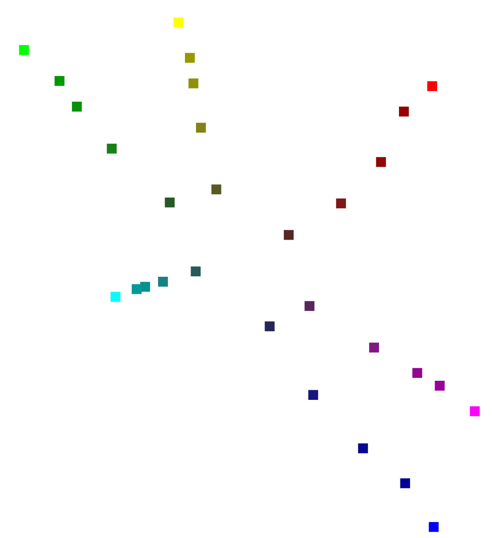
The border of the CIE La*b* color space is filled by colors which are saturated with high chroma. These are the colors in the outer rings of the HSB cylinder slice where the brightness is 1 and the saturation is 1.

RGB Values on CIE La*b* Color Space Border

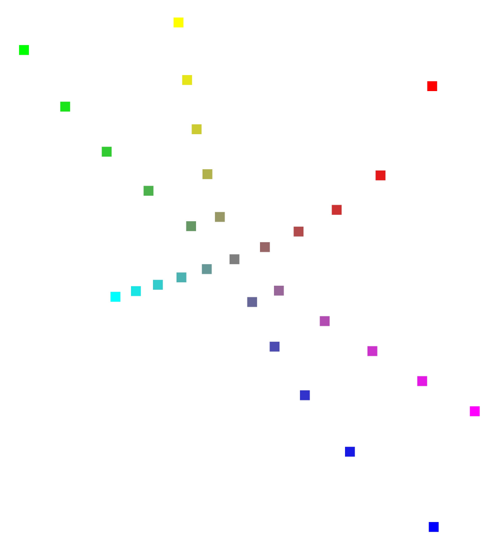


In any (R, G, B), with each coordinate having a different value, one of the three coordinates is the maximum of the three coordinates, one is a minimum, and one is in between.

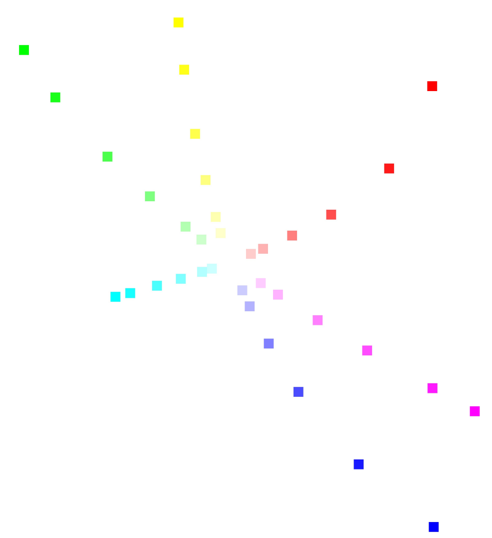
CIE La*b* Hues



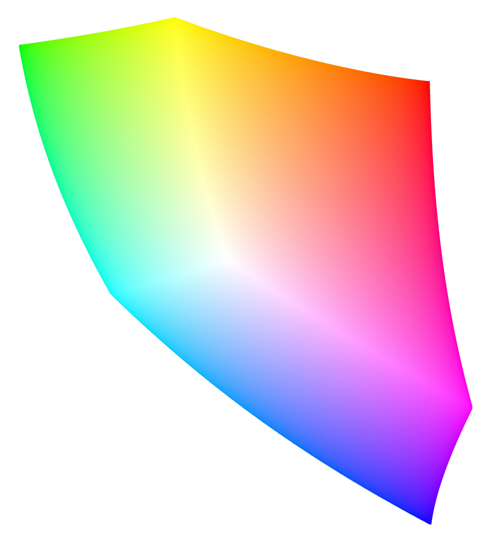
CIE La*b* Hues



CIE La*b* Hues

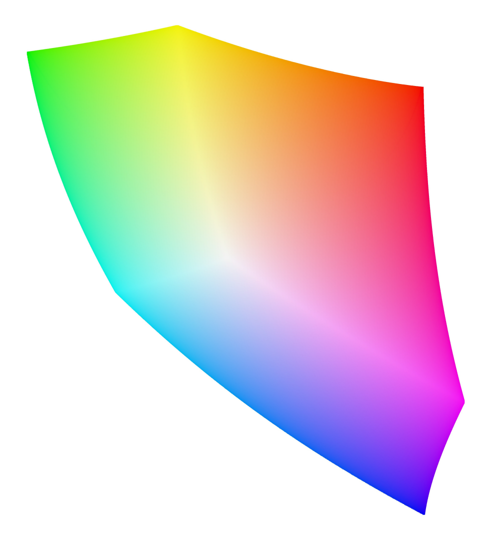


CIE La*b* Color Space



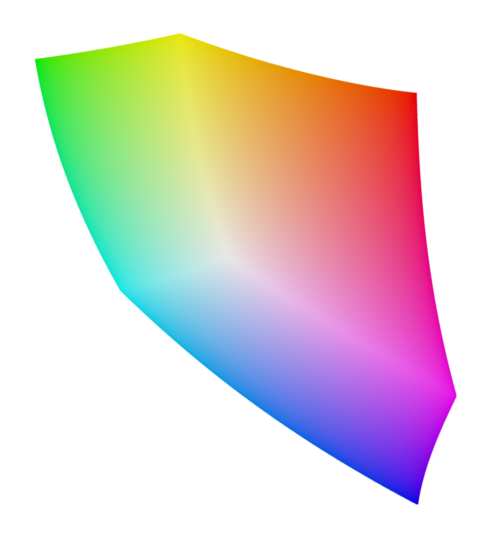
$$cmax = 1,\ cmin \in [0, 1]$$

CIE La*b* Color Space



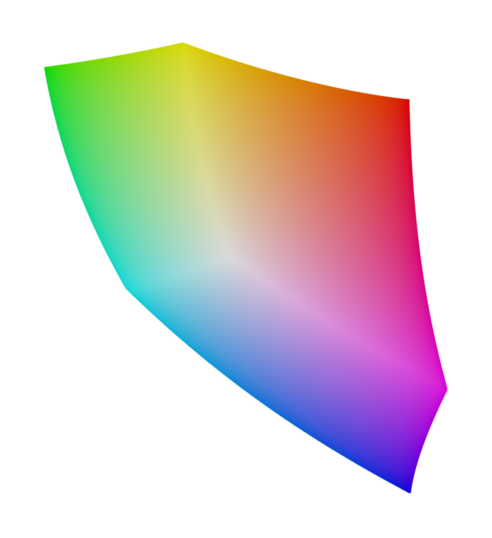
$$cmax = .9,\ cmin \in [0, cmax]$$

CIE La*b* Color Space



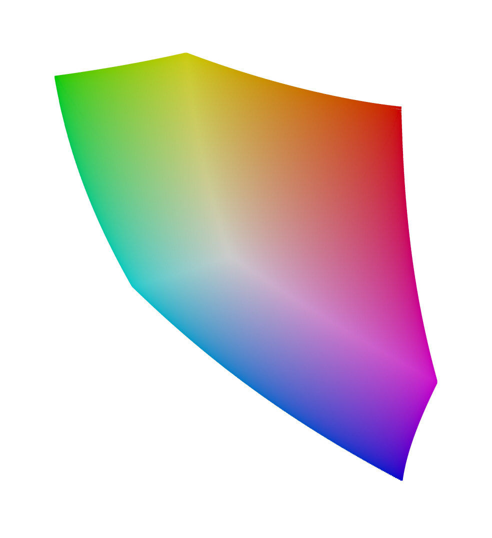
$$cmax = .8, \ cmin \in [0, cmax]$$

CIE La*b* Color Space



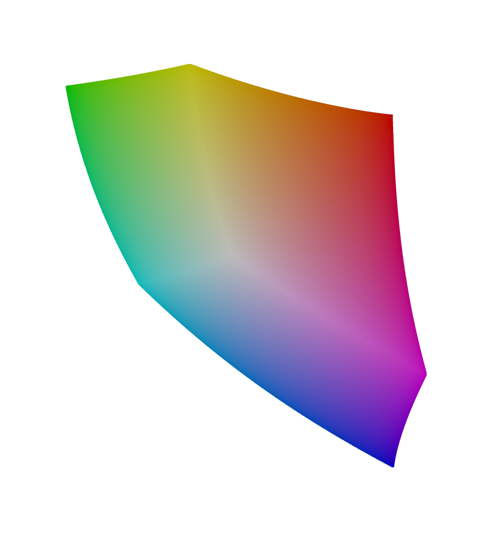
$$cmax = .7, \; cmin \in [0, cmax]$$

CIE La*b* Color Space



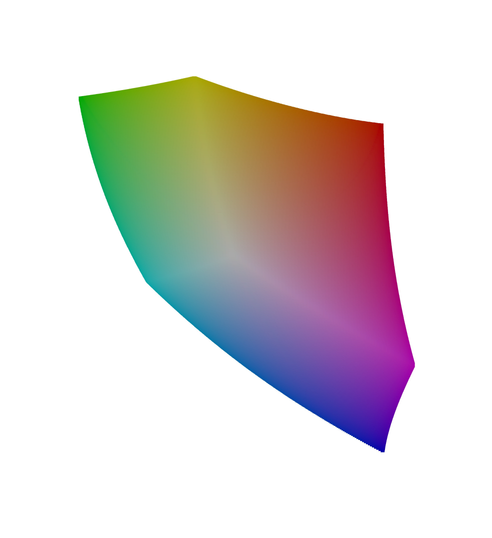
$$cmax = .6, \;\; cmin \in [0, cmax]$$

CIE La*b* Color Space



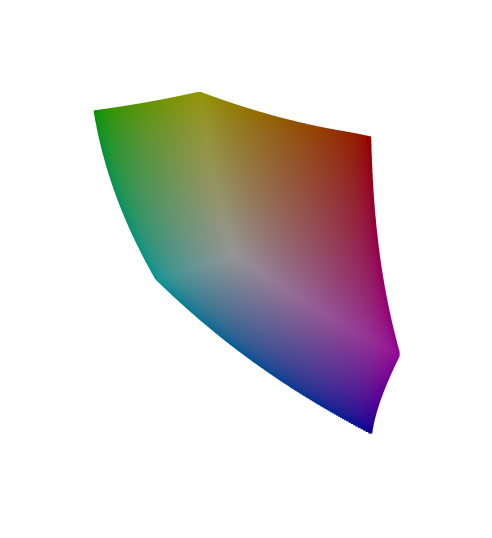
$$cmax = .5, \;\; cmin \in [0, cmax]$$

CIE La*b* Color Space



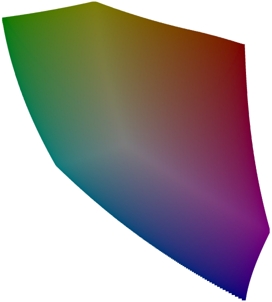
$$cmax = .4, \;\; cmin \in [0, cmax]$$

CIE La*b* Color Space



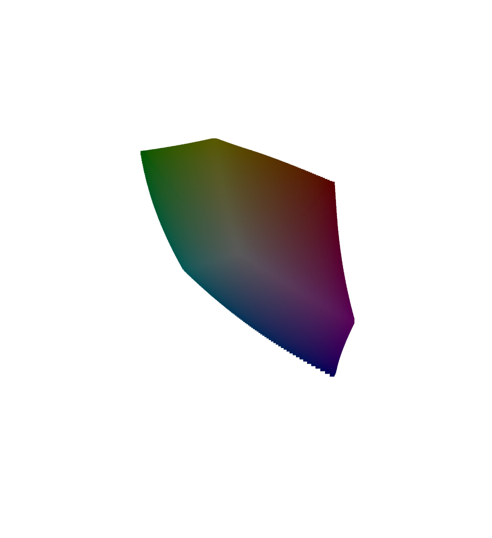
$$cmax = .3, \; cmin \in [0, cmax]$$

CIE La*b* Color Space



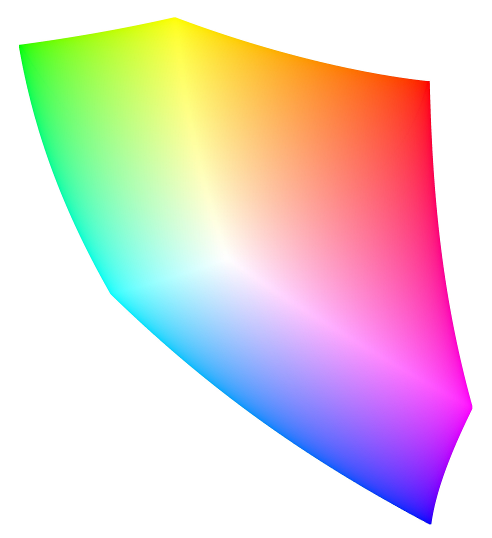
$cmax = .2, \ \ cmin \in [0, cmax]$

CIE La*b* Color Space



$cmax = .1, \; cmin \in [0, cmax]$

CIE La*b* Color Space



$$cmax = 1, \; cmin \in [0, cmax]$$

CIE La*b* Color Space

$$cmax = 1, \;\; cmin \in [.1, cmax]$$

CIE La*b* Color Space

$$cmax = 1, \ \ cmin \in [.2, cmax]$$

CIE La*b* Color Space

$$cmax = 1, \ \ cmin \in [.3, cmax]$$

CIE La*b* Color Space

$$cmax = 1, \;\; cmin \in [.4, cmax]$$

CIE La*b* Color Space

$$cmax = 1, \; cmin \in [.5, cmax]$$

CIE La*b* Color Space

$$cmax = 1, \ \ cmin \in [.6, cmax]$$

CIE La*b* Color Space

$$cmax = 1, \; cmin \in [.7, cmax]$$

CIE La*b* Color Space

$$cmax = 1, \;\; cmin \in [.8, cmax]$$

CIE La*b* Color Space

$$cmax = 1, \;\; cmin \in [.9, cmax]$$