The C Major Scale

Note	Freq	Ratio To C4	Ratio To C4	Approx. Frac.
C4	261.626	$2^{\frac{0}{12}}$	1.00000	1
D4	293.665	$2^{\frac{2}{12}}$	1.12245	$\frac{9}{8}$
E4	329.628	$2^{\frac{4}{12}}$	1.25992	$\frac{5}{4}$
F4	349.228	$2^{\frac{5}{12}}$	1.33484	$\frac{4}{3}$
G4	391.995	$2^{\frac{7}{12}}$	1.49831	$\frac{3}{2}$
A4	440.000	$2^{\frac{9}{12}}$	1.68179	$\frac{5}{3}$
B4	493.853	$2^{\frac{11}{12}}$	1.88775	$\frac{15}{8}$
C5	523.251	$2^{\frac{12}{12}}$	2.00000	2

A frequency that is twice the resonant frequency of a stretched wire will resonate on that stretched wire.

A frequency that is n times the resonant frequency of a stretched wire will resonate on that stretched wire.
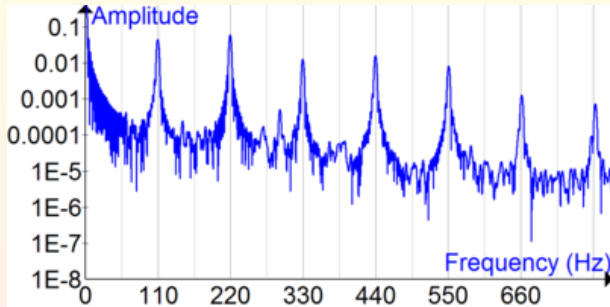
The fundamental frequency that stretched wire will resonate at will also resonate at its harmonics.

Look for the largest fraction that evenly divides the frequency of two notes.

C4	8/8	D4	9/8	1/8	C1
C4	4/4	E4	5/4	1/4	C2
C4	3/3	F4	4/3	1/3	F2
C4	2/2	G4	3/2	1/2	C3
C4	3/3	A4	5/3	1/3	F2
C4	8/8	B4	15/8	1/8	C1

C Major Chord C4 E4 G4

C4	4/4	E4	5/4	1/4	C2
C4	2/2	G4	3/2	1/2	C3

The C Minor Scale

Note	Freq	Ratio To C4	Ratio	Approx. Frac.
$C4$	261.626	$2^{\frac{0}{12}}$	1.00000	1
$D4$	293.665	$2^{\frac{2}{12}}$	1.12245	$\frac{9}{8}$
$E^{\flat}4$	329.628	$2^{\frac{3}{12}}$	1.18921	$\frac{6}{5}$
$F4$	349.228	$2^{\frac{5}{12}}$	1.33484	$\frac{4}{3}$
$G4$	391.995	$2^{\frac{7}{12}}$	1.49831	$\frac{3}{2}$
$A^{\flat}4$	466.164	$2^{\frac{8}{12}}$	1.58740	$\frac{8}{5}$
$B^{\flat}4$	493.853	$2^{\frac{11}{12}}$	1.88775	$\frac{15}{8}$
$C5$	523.251	$2^{\frac{12}{12}}$	2.00000	2

Look for the largest fraction that evenly divides the frequency of two notes.

C4	8/8	D4	9/8	1/8	C1
C4	5/5	$E^\flat$4	6/5	1/5	$A^\flat$1
C4	3/3	F4	4/3	1/3	F2
C4	2/2	G4	3/2	1/2	C3
C4	3/3	$A^\flat$4	5/3	1/3	F2
C4	8/8	$B^\flat$4	15/8	1/8	C1

C Minor Chord C4 E$^\flat$4 G4

C4	5/5	E$^\flat$4	6/5	1/5	A$^\flat$1
C4	2/2	G4	3/2	1/2	C3

0	Unison	1
1	Semitone	16/15
2	Whole Tone	9/8
3	Minor Third	6/5
4	Major Third	5/4
5	Perfect Fourth	4/3
6a	Augmented Fourth	45/32
6b	Diminished Fifth	64/35
7	Perfect Fifth	3/2
8	Minor Sixth	8/5
9	Major Sixth	5/3
10	Minor Seventh	9/5
11	Major Seventh	15/8
12	Octave	2/1

Pitch Classes

0	1	2	3	4	5	6	7	8	9	10	11	0
C	C#	D	D#	E	F	F#	G	G#	A	A#	B	C
C	D♭	D	E♭	E	F	G♭	G	A♭	A	B♭	B	C

Intervals

Major	2	2	1	2	2	2	1
Minor	2	1	2	2	2	2	1

Examples

C **Major**	C	D	E	F	G	A	B	C
E♭ **Major**	E♭	F	G	A♭	B♭	C	D	E♭
C# **Minor**	C#	D#	E	F#	G#	A#	C	C#

Major Key Signatures

0	1	2	3	4	5	6	7	8	9	10	11	0
C	C#	D	D#	E	F	F#	G	G#	A	A#	B	C
C	D♭	D	E♭	E	F	G♭	G	A♭	A	B♭	B	C

Major Scale Intervals		2	2	1	2	2	2	1

Name	Num ♭	Num #	1	2	3	4	5	6	7	1
C Major	0	0	C	D	E	F	G	A	B	C
G Major	0	1	G	A	B	C	D	E	F#	G
D Major	0	2	D	E	F#	G	A	B	C#	D
A Major	0	3	A	B	C#	D	E	F#	G#	A
E Major	0	4	E	F#	G#	A	B	C#	D#	E
B Major	0	5	B	C#	D#	E	F#	G#	A#	B
F# Major	0	6	F#	G#	A#	B	C#	D#	F	F#
G♭ Major	6	0	G♭	A♭	B♭	C♭	D♭	E♭	F	G♭
D♭ Major	5	0	D♭	E♭	F	G♭	A♭	B♭	C	D♭
A♭ Major	4	0	A♭	B♭	C	D♭	E♭	F	G	A♭
E♭ Major	3	0	E♭	F	G	A♭	B♭	C	D	E♭
B♭ Major	2	0	B♭	C	D	E♭	F	G	A	B♭
F Major	1	0	F	G	A	B♭	C	D	E	F

Minor Key Signatures

0	1	2	3	4	5	6	7	8	9	10	11	0
C	*C#*	*D*	*D#*	*E*	*F*	*F#*	*G*	*G#*	*A*	*A#*	*B*	*C*
C	*D♭*	*D*	*E♭*	*E*	*F*	*G♭*	*G*	*A♭*	*A*	*B♭*	*B*	*C*

Minor Scale Intervals	2	1	2	2	1	2	2

Name	Num ♭	Num #	1	2	3	4	5	6	7	1
A Minor	0	0	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*A*
E Minor	0	1	*E*	*F#*	*G*	*A*	*B*	*C*	*D*	*E*
B Minor	0	2	*B*	*C#*	*D*	*E*	*F#*	*G*	*A*	*B*
F# Minor	0	3	*F#*	*G#*	*A*	*B*	*C#*	*D*	*E*	*F#*
C# Minor	0	4	*C#*	*D#*	*E*	*F#*	*G#*	*A*	*B*	*C#*
G# Minor	0	5	*G#*	*A#*	*B*	*C#*	*D#*	*E*	*F#*	*G#*
D# Minor	0	6	*D#*	*F*	*F#*	*G#*	*A#*	*B*	*C#*	*D#*
E♭ Minor	6	0	*E♭*	*F*	*G♭*	*A♭*	*B♭*	*C♭*	*D♭*	*E♭*
B♭ Minor	5	0	*B♭*	*C*	*D♭*	*E♭*	*F*	*G♭*	*A♭*	*B♭*
F Minor	4	0	*F*	*G*	*A♭*	*B♭*	*C*	*D♭*	*E♭*	*F*
C Minor	3	0	*C*	*D*	*E♭*	*F*	*G*	*A♭*	*B♭*	*C*
G Minor	2	0	*G*	*A*	*B♭*	*C*	*D*	*E♭*	*F*	*G*
D Minor	1	0	*D*	*E*	*F*	*G*	*A*	*B♭*	*C*	*D*

Align the 0 with the tonic note of the chord

Major	0,4,7	C
Minor	0,3,7	C^{min}
Suspended	0,5,7	C^{sus}
Augmented	0,4,8	C^{aug}
Diminished	0,3,6	C^{dim}
Major Sixth	0,4,7,9	C^6
Minor Sixth	0,3,7,9	C^{min6}
Dominant Seventh	0,4,7,10	C^7
Major Seventh	0,4,7,11	C^{M7}
Minor Seventh	0,3,7,10	C^{min7}
Half Diminished Seventh	0,3,6,10	$C^{\phi 7}$
Diminished Seventh	0,3,6,9	$C^{\circ 7}$
Major Ninth	0,4,7,11,14	C^{M9}
Dominant Ninth	0,4,7,10,14	C^9
Dominant Minor Ninth	0,4,7,10,13	$C^{7\flat 9}$
Minor Ninth	0,3,7,10,14	C^{min9}

Consider the *C* augmented chord: 0,4,8.

0	4	8	C Augmented
4	8	0	C Augmented first inversion = E Augmented
8	0	4	C Augmented second inversion = G# Augmented

0	4	8	Augmented
4	8	0	Augmented first inversion
4	8	12	Augmented first inversion
0	4	8	Subtract 4 from each position: Augmented again

Consider the C^6 chord: 0,4,7,9

0	4	7	9	Major Sixth
4	7	9	0	Major Sixth First Inversion
4	7	9	12	Major Sixth First Inverstion
0	3	5	8	Not one of our named chords
7	9	0	4	Major Sixth Second Inversion
7	9	12	16	Major Sixth Second Inversion
0	2	5	9	Not one of our named chords
9	0	4	7	Major Sixth Third Inversion
9	12	16	19	Major Sixth Third Inversion
0	3	7	10	Minor 7th

C^6 Third inversion = A^{min7}

Consider the C^{min6} chord: 0,3,7,9

0	3	7	9	Minor Sixth
3	7	9	0	Minor Sixth First Inversion
3	7	9	12	Minor Sixth First Inversion
0	4	6	9	Not one of our named chords
7	9	0	3	Minor Sixth Second Inversion
7	9	12	15	Minor Sixth Second Inversion
0	2	5	8	Not one of our named chords
9	0	3	7	Minor Sixth Third Inversion
9	12	15	19	Minor Sixth Third Inversion
0	3	6	10	Half Diminished Seventh

C^{min6} Third Inversion = A Half Diminished Seventh

Minor 9th

$0 - 3 - 7 - 10 - 14$

D#

E

D

F

C#

F#

C

G

B

G#

A#

A
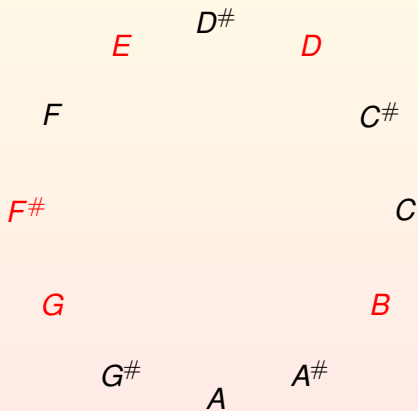
E Minor 9th

Normalized Chord

$$E \quad 4 \qquad D^{\#} \quad 3 \qquad D \quad 2$$

$$F \quad 5 \qquad\qquad\qquad 1 \quad C^{\#}$$

$$F^{\#} \quad 6 \qquad\qquad\qquad 0 \quad C$$

$$G \quad 7 \qquad\qquad\qquad 11 \quad B$$

$$8 \qquad 9 \qquad 10$$
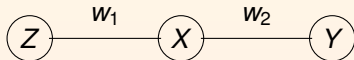
$$G^{\#} \qquad A \qquad A^{\#}$$
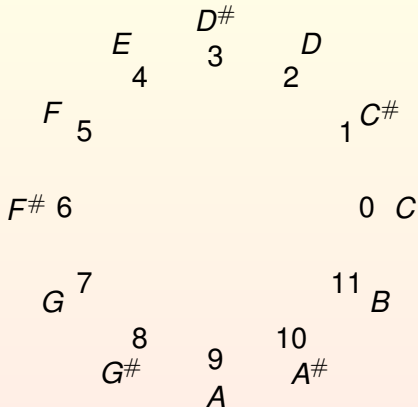
- Ignore the octave designation
- Replace letters by numbers modulo 12
  - $A4$ and $A5$ collapse to 9

$$\{D3, C4, A4, F5^{\#}, A5\} \rightarrow \{2, 0, 6, 9\}$$

Chord Distance Graph

- Nodes of chord distance graph
    - Pitch Equivalence Classes $\{0, \dots, 11\}$
    - Nonempty Subsets of $\{0, \dots, 11\}$ constitute 4095 subsets
    - 16 chords $\times$ 12 pitch equivalence classes = 192 named chords

- The neighbors of X are
- All subsets Z where
    - Z is X with one element deleted
- All subsets Y where
    - Y is X with one element changed by 1
- $CD(A, B)$: The Chord Distance between A and B
    - The length of the shortest path between A and B

Chord Distance

- $|X|$ is the cardinality of the set X
- $[N] = \{1, \ldots, N\}$
- w_1 cost of changing one note of a chord by a half step
- w_2 cost of removing one note of a chord

Let X and Y be normalized chords.
Without loss of generality we assume $|X| \geq |Y|$

$$
\begin{aligned}
f_m^-(\{x_1, \ldots, x_M\}) &= \{x_1, \ldots, x_{m-1}, (x_m - 1)_{mod12}, x_{m+1}, \ldots, x_M\}, \ m \in [|X|] \\
f_m^+(\{x_1, \ldots, x_M\}) &= \{x_1, \ldots, x_{m-1}, (x_m + 1)_{mod12}, x_{m+1}, \ldots, x_M\}, \ m \in [|X|] \\
g_m(\{x_1, \ldots, x_M\}) &= \{x_1, \ldots, x_{m-1}, x_{m+1}, \ldots, x_M\}), \ m \in [|X|]
\end{aligned}
$$

Let $CD(X, Y)$ be the chord distance between X and Y

$$
CD(X, Y) = \min \left\{
\begin{array}{l}
w_1 + \min_{m \in [|X|]} CD(f_m^-(X), Y) \\
w_1 + \min_{m \in [|X|]} CD(f_m^+(X), Y) \\
w_2 + \min_{m \in [|X|]} CD(g_m(X), Y)
\end{array}
\right.
$$

- Add or Subtract 1 semitone: weight 1
- Delete or add a note: weight 3

Find the shortest distance between $C^{\circ 7}$ and E^m

$$\rho((C, E^\flat, G^\flat, A), (E, G, B))$$

The shortest distance is 6. The C goes to the B, a change of -1. The $E^\flat$ goes to an E, a change of +1. The $G^\flat$ changes to a G a change of of +1. The A is deleted a change of 3.

- Nodes of chord distance graph
  - Pitch Equivalence Classes $\{0, \ldots, 11\}$
  - Nonempty Subsets of $\{0, \ldots, 11\}$ constitute 4095 subsets
  - 16 chords $\times$ 12 pitch equivalence classes = 192 named chords
- Given a non-empty subset of the pitch equivalence classes coming from the Midi music,
  - Find the closest named chords

Scale of the key signature

1	2	3	4	5	6	7
C	*D*	*E*	*F*	*G*	*A*	*B*
$A^\flat$	$B^\flat$	*C*	$D^\flat$	$E^\flat$	*F*	*G*

I	1,3,5	*CEG*	$A^\flat C E^\flat$
ii	2,4,6	*DFA*	$B^\flat D^\flat F$
iii	3,5,7	*EGB*	$C E^\flat G$
IV	4,6,1	*FAC*	$D^\flat F A^\flat$
V	5,7,1	*GBD*	$E^\flat G B^\flat$
vi	6,1,3	*ACE*	$F A^\flat C$
vii^ϕ	7,2,4	*BDF*	$G B^\flat D^\flat$

Scale of the key signature

1	2	3	4	5	6	7
C	*D*	*E$^\flat$*	*F*	*G*	*A$^\flat$*	*B$^\flat$*
F$^\#$	*G$^\#$*	*A*	*B*	*C$^\#$*	*D*	*E*

i	1,3,5	*C*	*E$^\flat$*	*G*	*F$^\#$*	*A*	*C$^\#$*
ii$^\phi$	2,4,6	*D*	*F*	*A$^\flat$*	*G$^\#$*	*B*	*D*
III	3,5,7	*E$^\flat$*	*G*	*B$^\flat$*	*A*	*C$^\#$*	*E*
iv	4,6,1	*F*	*A$\flat$*	*C*	*B*	*D*	*F$^\#$*
v	5,7,1	*G*	*B$^\flat$*	*D*	*C$^\#$*	*E*	*G$^\#$*
VI	6,1,3	*A$^\flat$*	*C*	*E$^\flat$*	*D*	*F$^\#$*	*A*
VII	7,2,4	*B$^\flat$*	*D*	*F*	*E*	*G$^\#$*	*B*

First Order Features

- Chords
  - Each scale has 7 notes
  - There are 14 kinds of chords
  - There are a total of $14 \times 7 = 98$ chords in the scale
  - For any midi piece, find the key signature
  - For any midi chord, find the closest ones in the set of 98
  - Determine the fraction of time each chord occurs
- Melody
  - Associate 0 with the tonic of the key signature
  - All other notes are encoded relative to the tonic
  - Assign to each note the number of semitones it is above the tonic in the C4 octave
  - Each sequence of 4 successive melody notes forms a measurement tuple

- Relative to Key Signature
  - Sequence of Melody Notes
  - Sequence of Bass Notes
  - Sequence of Chords
- Sequence of Durations

- Use $N = 4$ or $N = 5$
- N-Tuples of Successive Melody Notes
- N-Tuples of Successive Base Notes
- N-Tuples of Successive Chords
- N-Tuples of Successive Durations
- Count how many of each of the types of N-tuples
- Components of the Feature Vector are the normalized counts
- Use a linear decision rule

Tuple Features

- L set of notes, pitch classes or chords or intervals
- $< x_1, \ldots, x_N >$, $x_n \in L^K$ observed sequence of K-tuples
- $h : L^K \rightarrow \mathbb{R}$ feature vector
    - Indicates for each K-tuple its probability of occurring

$$h(y) = \frac{|\{n \mid x_n = y\}|}{N - K + 1}$$

Definition

The interval from a pitch class *m* to pitch class *n* is the smallest absolute number of semitones that have to be added to or subtracted from *m* to get to *n*.

m	n	interval
0	1	1
1	11	-2
4	6	2
10	7	-3
2	10	-4

$$i = \left\{ \begin{array}{ccc} n - m & \text{if} & -6 < n - m \leq 6 \\ |n - m| - 12 & \text{if} & |n - m| > 6 \end{array} \right.$$