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Understanding Engineering Drawings

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It is shown that understanding the three dimensional objects depicted in the orthographic views employed in engineering drawings can be achieved by solving a sequence of three consistent labeling problems. The constraints used in the engineering drawings are stated, the translation to the constraints of the consistent labeling problems are given, and the computer solution is illustrated by the execution of an appropriate tree search.

1. INTRODUCTION

The world consists of objects having three dimensions: height, width, and depth. Drawings and pictures are two-dimensional representations of these three-dimensional objects. One system of representation of three-dimensional objects in two-dimensional drawings is the orthographic views employed in engineering drawings. These drawings are universally used in manufacturing because they do not have perspective distortion and they facilitate inspection which can be easily done by comparing the manufactured part with a full-size drawing in a template-matching manner.

In this paper an algorithm is developed to enable a computer to understand the three-dimensional objects represented in engineering drawings. By understanding we mean being able to give the surface equation of every face and the equations of the bounding-line segments to every face on the basis of the line segments in the engineering drawing.

Three-dimensional objects are represented in engineering drawings by two to six two-dimensional orthogonal views. The American standard arrangement for the six principal views is shown in Fig. 1 (1).

The following definitions are used in reference to Fig. 1 and in the remainder of the paper.

1. The line of sight of a view is the direction from which the object is viewed.
2. Any two views placed side-by-side are called adjacent views and have their common dimensions aligned.
3. The parallel lines connecting and aligning adjacent views are called parallels.
4. All views adjacent to the same view are called related views.

In Fig. 1 the top and front views are adjacent views, but the right side and top views are related views.

The orthographic geometry has the following properties.

Rule 1. The lines of sight for any two adjacent views are perpendicular.

Rule 2. Every point of the object in one view is aligned on a parallel directly opposite the corresponding point in any adjacent view.

Rule 3. The distance between any two points on the object measured along the parallels is the same in all related views.

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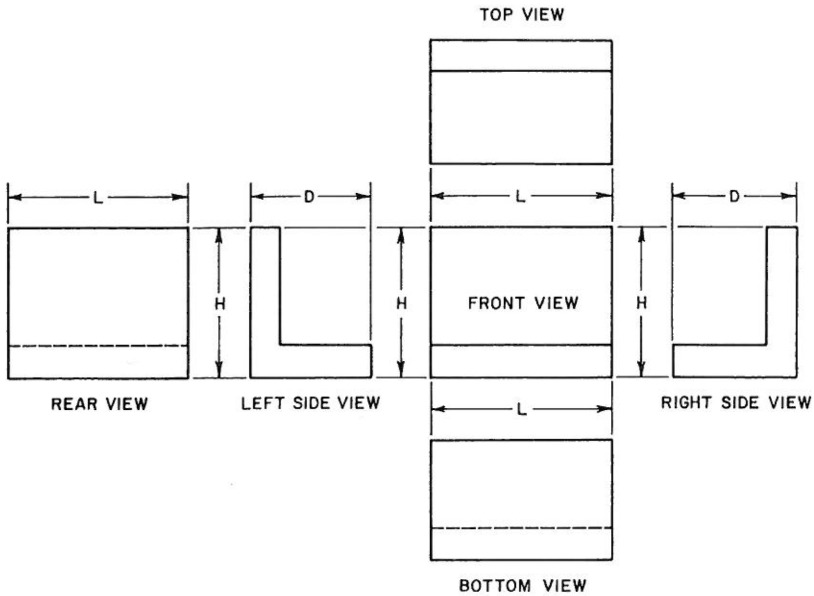

FIG. 1. The American standard arrangement for the six principal views.

From the orthographic geometry it is apparent that each view contributes information not in the other views and that to understand the object portrayed by the orthographic view the information in one view must be used in a coordinated way with the other views. In the remainder of this section the coordinating process for three-dimensional objects with planar faces is explained.

The analysis of the object consists of considering the object as three sets of its components parts: a set of points, a set of lines, and a set of faces. Rules 2 and 3 give the constraint relationship for the set of points.

To develop the relationships for the set of lines and the set of faces, the principal views of a line and a triangular plane are given in Figs. 2 and 3, respectively.

As seen in Fig. 2 a line is seen as a line or as a point that is an end view of the line. The fourth rule can thus be given.

Rule 4. A line can only appear as a line or a point, a point being the end view of a line.

As in Fig. 3, the face is seen as a face with the same number of vertices or as a line, the line being an edge view of the plane. The fifth rule can thus be given.

Rule 5. Every face can appear only as an edge or as a figure of similar configuration.

To develop the relationships for the set of faces, an understanding of the views must be developed. First, a line is understood as being the intersection of two faces. Therefore, the two faces intersecting in this line must be in different planes. Second, the top view shows the highest observable points and faces of the object. Similarly, the most extreme front, lateral, back, and bottom points and faces are seen from the

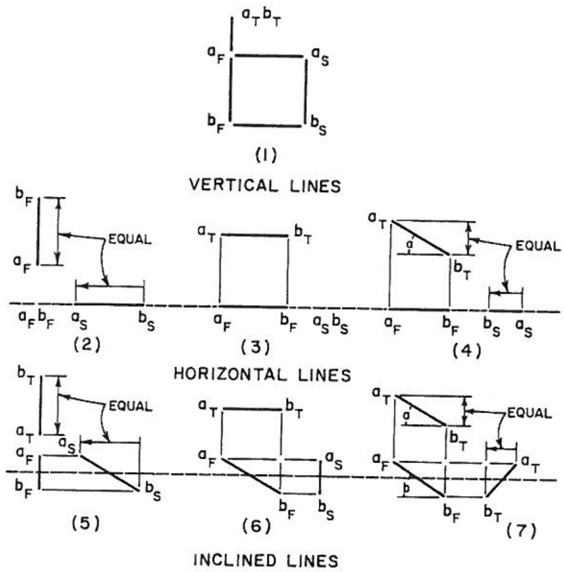

FIG. 2. The principal views of vertical, horizontal, and inclined lines.

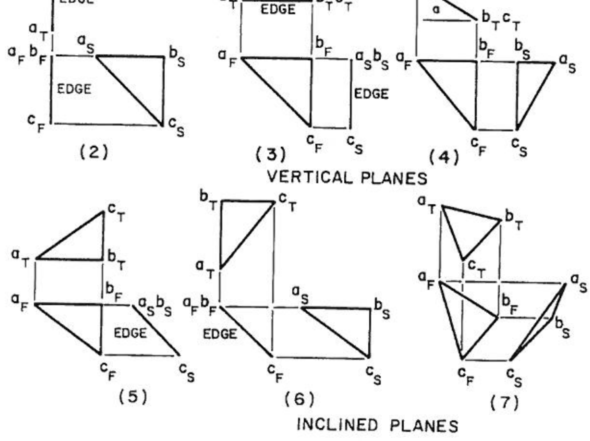

FIG. 3. The principal views of horizontal, vertical, and inclined triangular planes.

front view, side view, back view, and bottom view, respectively. Using these two principles, the last two rules, called the view consistency rules, can be given.

Rule 6. Each face seen in a view is the first face seen along the line of sight for that view.

Rule 7. No two contiguous faces can lie in the same plane.

These seven rules are the constraints needed to understand an engineering drawing where understanding means to be able to precisely describe the three-dimensional objects given in the two-dimensional orthographic views. In Section 2 we show one method of understanding engineering drawings. In Section 3 we show how understanding the engineering drawing is a consistent labeling problem.

2. A METHOD FOR UNDERSTANDING ENGINEERING DRAWINGS

The process of understanding an engineering drawing of an object means being able to identify each face of the object and the (x, y, z) coordinates of each vertex of each face. To do this requires a tree search given the original two-dimensional coordinates on the engineering drawing. In this section we discuss what information is selected from the engineering drawing and how it is used in a tree search.

The tree search has four parts: first, to determine the set of (x, y, z) three-dimensional points eligible to be vertices; second, to determine the set of visible surfaces; third, to determine for each visible surface a sequence of eligible three-dimensional vertices that is consistent with the observed view of the surface; and fourth, to select from the available vertex sequences one sequence, called an interpretation, for each visible surface face that is consistent with that interpretation selected for every other face.

The set of eligible vertices is easily determined. Let P be the set of vertex points given in the top view, Q the set of vertex points given in the front view, and R the set of vertex points specified on the side view. Each point p in P has x and y coordinates which we denote by $x(p)$ and $y(p)$. Each point q in Q has x and z coordinates, which we denote by $x(q)$ and $z(q)$. Each point r in R has y and z coordinates which we denote by $y(r)$ and $z(r)$. Let V denote the set of eligible vertices. We shall specify each point in V by specifying a triple (p, q, r) from the top, front, and side views:

$$V = \{(p, q, r) \in P \times Q \times R \mid x(p) = x(q),\ y(p) = y(r),\ z(q) = z(r)\}.$$

This is a restatement of Rules 2 and 3. Obviously, the x coordinate of any triple in V can be obtained as an x coordinate of its first or second component. The y coordinate of any triple in V can be obtained as the y coordinate of its second or third component. The z coordinate of any triple in V can be obtained as the z coordinate of its first or third component.

The set of projections of visible surfaces is determined next. The following terms are used in explaining how the projections of the visible surfaces are determined.

1. The valency of a point p is the number of lines in the object having the point p for one of its endpoints.

2. A normalized line is a line with unit length. For a line passing through $(0, 0)$ and (x, y), the normal representation would be $(x/\sqrt{x^2 + y^2},\ y/\sqrt{x^2 + y^2})$.

3. To determine the right-most (left-most) line at a given point, set up a new coordinate system with the point at $(0, 0)$ and the last line traveled as the negative x

axis. Then there are three cases for determining the right-most (left-most) line, given two lines.

In the new coordinate system observe the following.

1. If both normalized lines have positive second coordinates, choose the line with the larger (smaller) first coordinate.
2. If both normalized lines have negative second coordinates, choose the line with the smaller (larger) first coordinate.
3. If the lines have second coordinates of different signs, choose the line with the negative (positive) second coordinate.

Each projection surface is specified as a sequence of two-dimensional vertices from one of the views. For a given view, we begin at an arbitrary starting point. For example, the starting point can be chosen from the set of points with the smallest first coordinate and second coordinate, taken in that order. From the starting point we travel along its leftmost line to the next point, using the line, in normal representation $(1, 0)$, as the last line traveled. We continue to travel along the rightmost line at each point until the starting point is reached. The area enclosed by the boundary we just traveled around is then given a unique label. Take as the next starting point the first point reached with a valency of 3 or greater. Reduce the valency of each point by 1 each time it is used in a surface. An example of determining the visible surfaces is given in Section 3.

Let $\langle p(0), \ldots, p(I-1) \rangle$ be the sequence of vertices of a visible surface seen in top view. The sequence $S = \langle (p(0), q(0), r(0)), \ldots, (p(I-1), q(I-1), r(I-1) \rangle$ is called an interpretation of the surface whose top-view projection is given by $\langle p(0), \ldots, p(I-1) \rangle$ if and only if

1. $(p(i), q(i), r(i)) \in V$, $i = 0, \ldots, I-1$;
2. there exists a line in the top view whose endpoints are given by $(p(i), p(i+1))$;
3. there exists a line in the front view whose endpoints are given by $(q(i), q(i+1))$; and
4. there exists a line in the side view whose endpoints are given by $(r(i), r(i+1))$.

(Note that the index arguments are taken modulo I. Also, it is legal, by Rule 4, for a line to be seen on end in some view. In this case both endpoints are identical.)

Likewise, the sequence S is called an interpretation of the front view visible surface whose projection is given by $\langle q(0), \ldots, q(I-1) \rangle$ if and only if S satisfies 1–4. And the sequence S will be called an interpretation of the side-view visible surface whose projection is given by $\langle r(0), \ldots, r(I-1) \rangle$ if and only if S satisfies 1–4.

Finally, we restate the criteria for consistent interpretation of surfaces: top-view surfaces must be higher (must have points with larger y coordinates); front-view surfaces must be nearer (must be points with smaller z coordinates); side-view surfaces must be points with larger x coordinates; and two object surfaces in the same view must lie in different planes. If S_1 is an interpretation of any surface visible from the top view and S_2 is an interpretation of any surface visible from another view, then to be consistent $(x, y, z) \in S_1$ and $(x, y, z') \in S_2$ implies $z > z'$. If S_1 is an interpretation of any surface visible from the front view and S_2 is an interpretation of any surface visible from another view, then to be consistent $(x, y, z) \in S_1$ and $(x, y', z) \in S_2$ implies $y > y'$. If S_1 is an interpretation of any surface visible from the side view and S_2 is an interpretation of any surface visible from another view, than to be consistent $(x, y, z) \in S_1$ and $(x', y, z) \in S_2$ implies $x > x'$.

EXAMPLE. Consider the object of Fig. 4a. It can be represented by three views, one from each of the directions illustrated by the arrow in the figure. These three views are shown in Fig. 4b where the respective coordinate system for each view is given. Notice that in Fig. 4b we have labeled each vertex in each view with a unique numeric label.

The set of points P, Q, R in the top, front, and side views are given by

$$P = \{1, 2, 3, 4\}$$
$$Q = \{5, 6, 7, 8, 9\}$$
$$R = \{10, 11, 12, 13, 14\}.$$

The set V of triples eligible to be vertices of the object is, therefore,

$$V = \{(1,5,11), (1,7,14), (2,6,11), (2,8,14), (3,5,10), (3,7,13), (4,6,10), (4,9,12), (4,8,13)\}.$$

In Fig. 4b there are six visible surfaces in the three views. In Fig. 4c, we have labeled the areas of the surface projections by the letters A, B, C, D, E, and F. To obtain a sequence of eligible vertices for a surface projection, we note that for any two points in a sequence of eligible vertices, there must be a line between each pair of corresponding points from the same view. Noting also from Rule 4 that a line

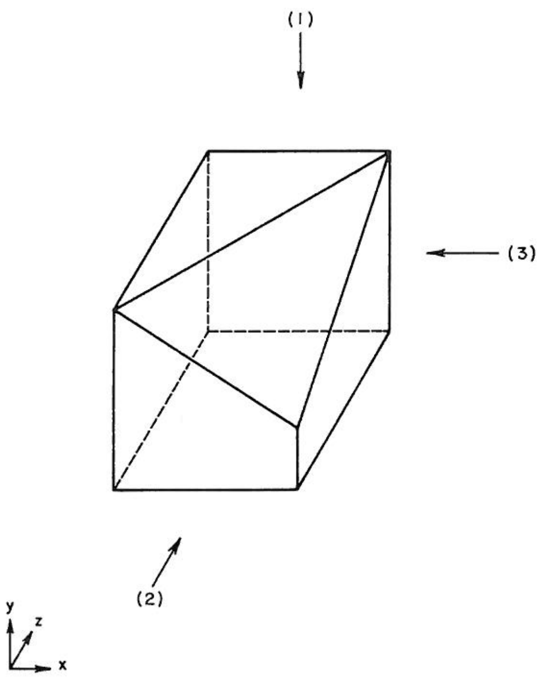

FIG. 4a. A three-dimensional object.

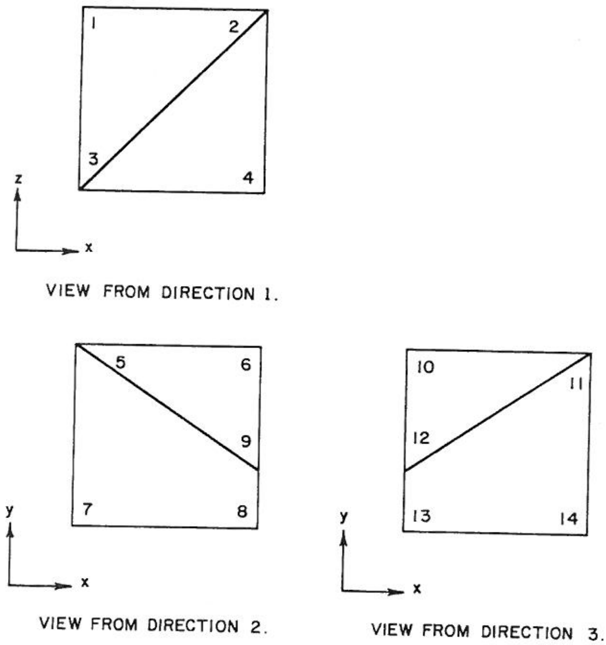

FIG. 4b. The three views of a three-dimensional object specified in the engineering drawing of Fig. 4a.

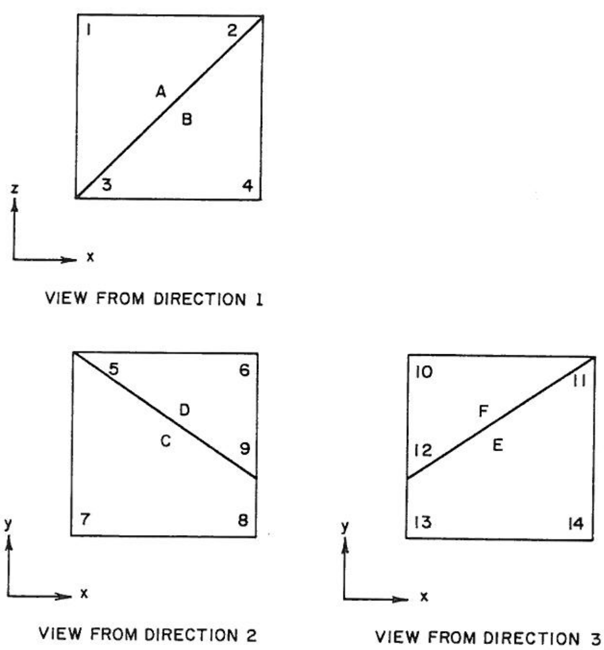

FIG. 4c. The three views of a three-dimensional object specified in the engineering drawing of Fig. 4a with the visible surfaces labeled.

TABLE 1

Sequences of Eligible Vertices for the Projection of Each Visible Surface

Surface	Interpretation
A	$S_1 = \langle(1,5,11),(2,6,11),(3,5,10)\rangle$
	$S_2 = \langle(1,7,14),(2,8,14),(3,7,13)\rangle$
B	$S_3 = \langle(2,6,11),(3,5,10),(4,6,10)\rangle$
	$S_4 = \langle(2,6,11),(3,5,10),(4,9,12)\rangle$
	$S_5 = \langle(2,8,14),(4,8,13),(3,7,13)\rangle$
C	$S_6 = \langle(3,5,10),(4,9,12),(4,8,13),(3,7,13)\rangle$
D	$S_7 = \langle(3,5,10),(4,6,10),(4,9,12)\rangle$
	$S_8 = \langle(3,5,10),(2,6,11),(4,9,12)\rangle$
E	$S_9 = \langle(2,6,11),(4,9,12),(4,8,13),(2,8,14)\rangle$
F	$S_{10} = \langle(2,6,11),(4,9,12),(4,6,10)\rangle$
	$S_{11} = \langle(2,6,11),(4,9,16),(3,5,10)\rangle$

may appear as an end view, we see that Table 1 gives all the interpretations for each visible surface.

To obtain a consistent interpretation we note that in any pair of interpretations, adjacent surfaces from the same view must not lie in the same plane and that any pair of interpretations in different views must satisfy the constraint that top-view surfaces must be higher, front-view surfaces must be nearer, and side-view surfaces must have greater x coordinates. Two interpretations of the object are shown in Fig. 5.

3. THE UNDERSTANDING OF AN ENGINEERING DRAWING AS A CONSISTENT LABELING PROBLEM

The consistent labeling problem is a generalization of the constraint satisfaction problem (2). The labeling problem involves a set of units, a set of labels, the constraint relation for the given pairs of units, and units and labels. The consistent labeling problem is to find a label for each unit such that the resulting set of unit–label pairs is consistent with the constraints. More formally, if U is the set of units and L is the set of labels, then the binary constraint R can be represented as a binary relation on $U \times L$: $R \subseteq (U \times L) \times (U \times L)$. If a pair of unit labels $(u_1, l_1, u_2, l_2) \in R$, then labels l_1, and l_2 are said to be consistent for units u_1, and u_2. A labeling f of all the units is called a consistent labeling if for every pair u_1, u_2 of units $(u_1, f(u_1), u_2, f(u_2)) \in R$. In this section we shall show why understanding an engineering drawing is a consistent labeling problem.

The understanding of an engineering drawing has four parts: first, to determine the set of (x, y, z) coordinates eligible to be vertices; second, to determine the set of projections of visible surfaces; third, to determine for each projection of a visible surface a sequence of eligible vertices; and fourth, selecting from the available vertex sequences an interpretation for each surface face that is consistent with that interpretation selected for every other face. The first, third, and fourth parts are solvable using the consistent labeling problem. The second part is solvable using the algorithm of Section 2.

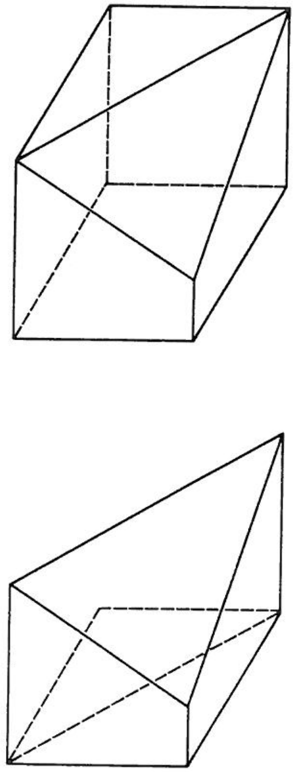

FIG. 5. Two of the three possible interpretations for the engineering drawing of Fig. 4b. The top object is a box with its upper right-hand corner cut off. The bottom object shows a box with its upper right-hand corner cut off and its back left-hand corner cut off all the way to the base, which has a thin triangular sheet extending to the back bottom left-hand corner.

The first part: *to determine the set of* (x, y, z) *coordinates eligible to be vertices*. Let the set of units be the views of the object, $U = \{v_1, \ldots, v_6\}$. Let the set of labels be the set of points from the two-dimensional drawings, $L = \{p_1, \ldots, p_n\}$. Then for a labeling $(p_1, p_2, \ldots p_6)$ we have the following constraints.

1. p_1 must be in view v_1.
2. p_2 must be in view v_2.

$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$

6. p_6 must be in view v_6.
7. The common coordinate for points p_i and p_j must be equal.

Then the unit relation T and the unit–label relation R can be defined using 1–7.

1. $(u_i, u_j) \in T$ if u_i and u_j are different views.
2. $(u_i, l_i, u_j, l_j) \in R$ if the common coordinate of points l_i and l_j is equal.

The third part: *To determine for each projection of a visible surface a sequence of eligible vertices*. Let the set of units be the points in the projection $U = \langle p(0), \ldots,$

$p(I-1)\rangle$ of the surface. Let the set of labels be the set of eligible vertices $L = (p_1, p_2, \ldots, p_6)$. Then we have the following constraints on the labeling from U to L.

1. $(p_1(i), p_1(i+1))$ is a line in view v_1.
2. $(p_2(i), p_2(i+1))$ is a line in view v_2.

$\vdots$

6. $(p_6(i), p_6(i+1))$ is a line in view v_6.

Then the unit relation T and the unit–label relation R can be defined using 1–6.

1. $(u_i, u_j) \in T$ if u_i and u_j are points in surface projection S.

2. $(u_i, l_i, u_j, l_j) \in R$ if (1) there exists a line between all corresponding points from the same view in the eligible vertices l_i and l_j when u_i and u_j are consecutive points in the surfaces, or (2) if u_i, u_j are not consecutive points on the projection of the visible surfaces.

The fourth part: *To detect from the available vertex sequences an interpretation for each surface segment that is consistent with that interpretation selected for every other segment*. Let the set of units be the set of visible surfaces, $U = \{A, B, \ldots, N\}$. Let the set L of labels be the set of surfaces generated by the third part, $L = \{S_1, \ldots, S_m\}$. Then we have the following constraints on the labeling from U to L.

1. Top view surfaces must be higher.
2. Front view surfaces must be nearer.
3. Side view surfaces must be farther to that side
4. Bottom view surfaces must be lower.
5. Back view surfaces must be farther away.
6. Any two contiguous surfaces must be in different planes.

Then the unit relation T and the unit–label relation R can be defined using 1–6.

1. $(u_i, u_j) \in T$ if $i \neq j$. That is, u_i and u_j are different projections of a visible surface.

2. $(u_i, l_i, u_j, l_j) \in R$ if the sequences of eligible vertices, l_i and l_j, satisfy 1–6 for their respective views.

The example of Section 2 is now redone using a tree search. The point sets and line sets are given for the object in Tables 2 and 3, respectively.

To select the set of eligible vertices, we construct the graph of Fig. 6 showing all pairs of inconsistent points. Treating this as a binary constraint, we may use the forward checking algorithm to determine all the consistent vertices. Part of the tree search for the eligible vertices are shown in Fig. 7.

An example of finding the visible surfaces for the view seen from direction 1 in Fig. 4a is as follows.

The starting point is the point $(0,0)$, which is point 3. The last line traveled is the normalized line $(1,0)$. There are three lines—lines 2, 4, and 5—from which to choose the leftmost line. As seen in Table 3, all three lines have a positive second coordinate. The one with the smallest first coordinate is chosen. This is line 2 which

TABLE 2
The Set of Points

Point	X	Y
1	0	3
2	3	3
3	0	0
4	3	0
5	0	3
6	3	3
7	0	0
8	3	0
9	3	1
10	3	0
11	3	3
12	1	0
13	0	0
14	0	3

goes from point 3 to point 1. At point 1 the only line to choose is line 1 which goes from point 1 to point 2. At point 2 there are two lines—lines 3 and 5—from which to choose the right-most line. As seen in Table 3 both lines have a negative second coordinate. The one with the smallest first coordinate is chosen. This is line 5 which goes from point 2 to point 3. At this point an area is finished. This area is labeled area A. The next area is now found.

TABLE 3
The Set of Lines

Line	First point	Second point	Normal representation
1	1	2	(1.0000, 0.0000)
2	1	3	(0.0000, − 1.0000)
3	2	4	(0.0000, − 1.0000)
4	3	4	(1.0000, 0.0000)
5	2	3	(−0.7070, −0.7070)
6	5	6	(1.0000, 0.0000)
7	5	9	(0.8320, − 0.5547)
8	5	7	(0.0000, − 1.0000)
9	6	9	(0.0000, − 1.0000)
10	7	8	(1.0000, 0.0000)
11	8	9	(0.0000, 1.0000)
12	10	11	(1.0000, 0.0000)
13	10	12	(0.0000, − 1.0000)
14	11	12	(−0.8320, −0.5547)
15	11	14	(0.0000, − 1.0000)
16	12	13	(0.0000, − 1.0000)
17	13	14	(1.0000, 0.0000)

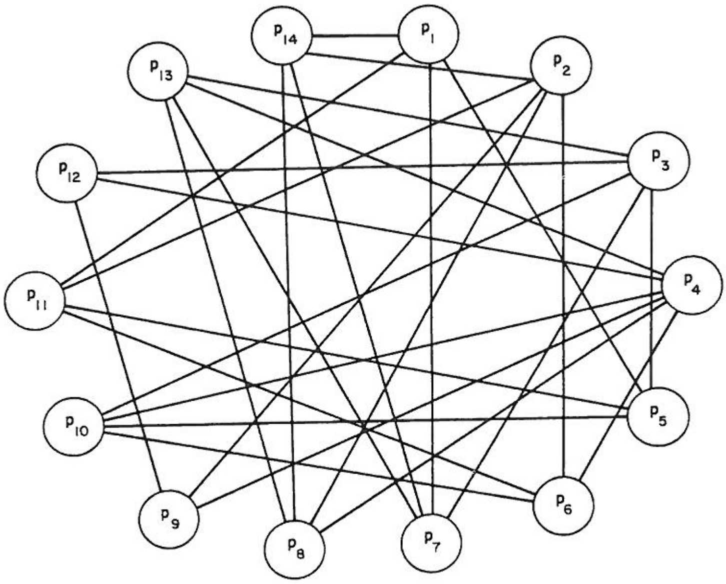

FIG. 6. All pairs of points that are compatible.

Point 3 is chosen as the starting point since in area *A* it is the first point reached with a valency of 3 or greater. The last line traveled is the normalized line (1,0). There are two lines—lines 4 and 5—from which to choose the leftmost line at point 3 since line 2 is eliminated when the valency is reduced after line 2 is used in area *A*. Since both lines have a positive second coordinate, the line with the smaller first coordinate is chosen. This is line 5 which goes from point 3 to point 2. At point 2

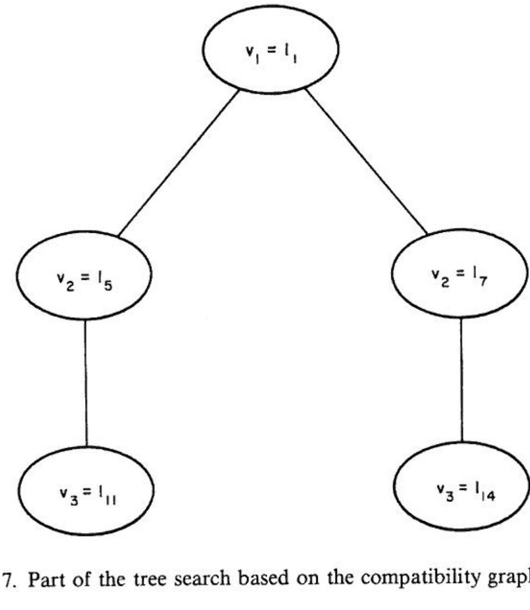

FIG. 7. Part of the tree search based on the compatibility graph of Fig. 6.

there are two lines—lines 1 and 3—from which to choose the rightmost line. Since line 5 is not on the negative x axis with respect to point 2, the normalized lines must be redefined for the new coordinate system:

$$\text{Line 1} \quad (-0.7070, 0.7070)$$
$$\text{Line 3} \quad (-0.7070, -0.7070).$$

Since the lines have different second coordinates, the one with the negative second coordinate is chosen. This is line 3 which goes from point 2 to point 4. At point 4 the only line to choose is line 4 which goes from point 4 to point 3. At this point an area is finished and this area is labeled area B. There are no points in area B with valency three or greater, so this view is finished.

To select a sequence of eligible vertices for each visible surface, we construct the graph of Fig. 8 showing all pairs of consistent vertices. Treating this as a binary

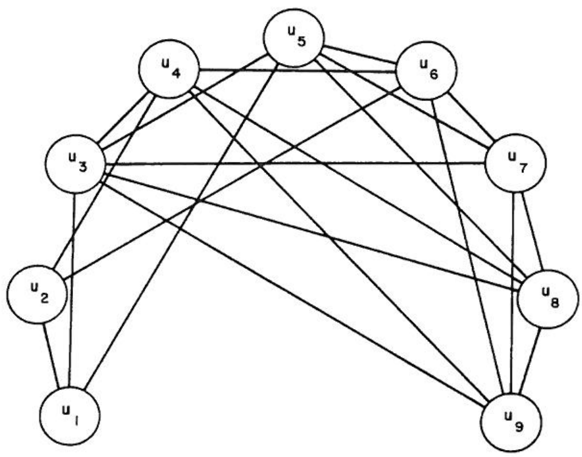

FIG. 8. All pairs of eligible vertices that are compatible.

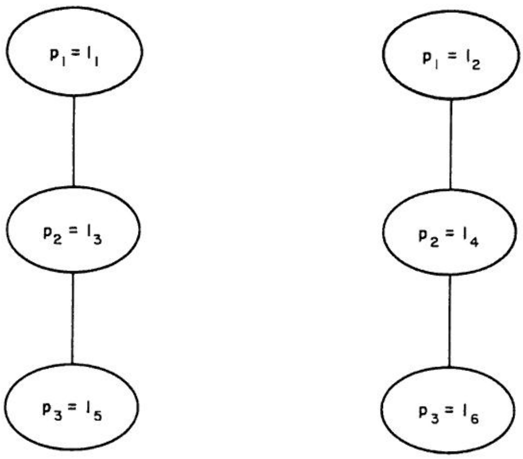

FIG. 9. Part of the tree search based on the compatibility graph of Fig. 8.

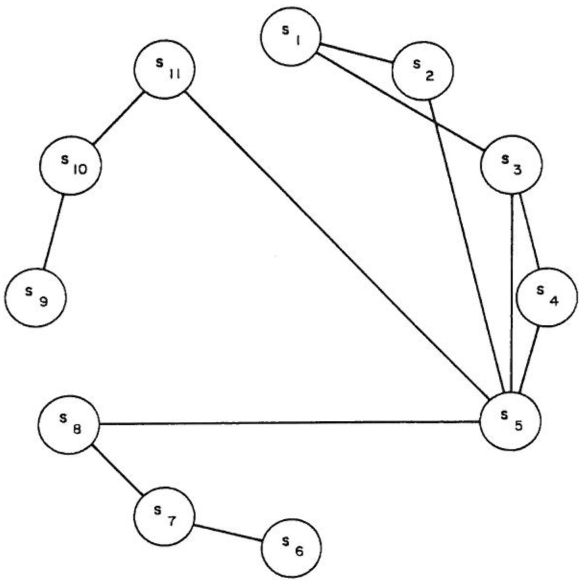

FIG. 10. All pairs of interpretations that are incompatible.

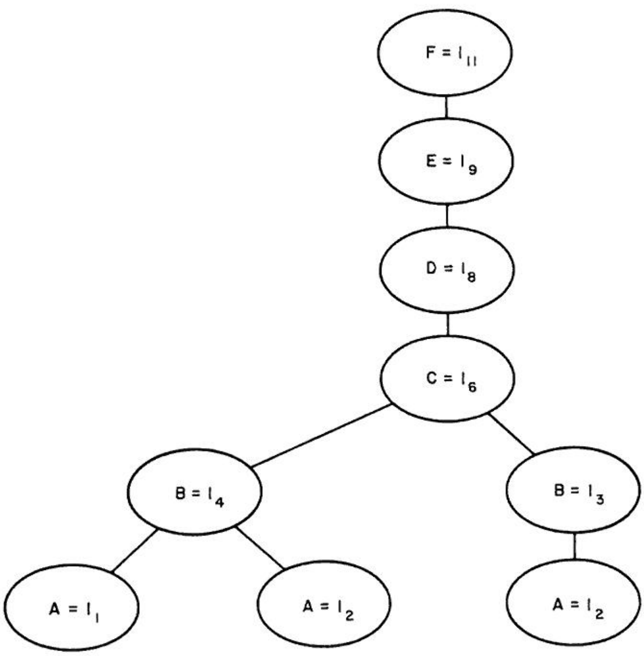

FIG. 11. The tree search based on the incompatibility graph of Fig. 10 and using the forward checking algorithm (3).

constraint, we may use the forward checking algorithm (3) to determine all the sequences of the eligible vertices. Part of the tree search for the sequences of eligible vertices is shown in Fig. 9.

To select a vertex sequence for each surface that is consistent with the interpretation for every other segment, we construct the graph of Fig. 10 showing all the pairs of inconsistent interpretations. Treating this as a binary constraint, we may use the forward checking algorithm (3) to determine all the consistent interpretations. A tree search for the interpretations is shown in Fig. 11.

4. CONCLUSION

In this paper we have shown that understanding an engineering drawing amounts to solving three consistent labeling problems: one to obtain a set of eligible vertices, one to obtain a set of candidate vertex sequences, and one to obtain the set faces. We have translated the interpretation rules employed in understanding engineering drawings to the constraints of the consistent labeling problem and have illustrated the solution to one example problem.

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