An Analog Method for Character Recognition*

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Summary—A method for character recognition which is capable of an analog implementation has been studied by simulation on a digital computer. In essence, this method involves maximizing the cross-correlation value between the unknown character and a set of average characters, there being one average character for each allowed character class. An average character is represented by a two-dimensional function. The value of this function at a point is the probability of occurrence of a mark at that point for the character class represented by the average character. Negative weights are given to areas of low probability in each average character to improve discriminability.

The simulation results indicate that this method is applicable to the recognition of machine printing, and perhaps to the recognition of constrained hand printing. The method can be implemented in an economical manner using electro-optical techniques.

Introduction

By and large, the pattern-recognition machines of today, whether in use or merely proposed, are digital in nature. That is, the pattern to be recognized is generally quantized in both position and density before any of the procedures for recognition are applied. The recognition procedure is generally then implemented by using binary logical circuitry. There are several advantages that might be gained if the pattern and all pertinent or derived information is kept in an analog form[1,2] for as long as possible. Prominent among these advantages are low cost and lack of quantizing error. In addition, a speed advantage may sometimes be realized.

A particular method for character recognition which is capable of an analog implementation has been studied by simulation on the IBM 704 digital computer. The results of the simulation indicate that this method is applicable to the recognition of machine printing, and perhaps to the recognition of constrained hand printing. The method can be implemented with simple optics for the most part, yielding an economic machine.

This method of character recognition is described in this paper. Simultation parameters and results are presented, and a means for optically implementing the method is discussed.

Description of Method

The character recognition method to be described depends upon a set of average characters, there being one average character for each of the allowable character classes. (A character class is the collection of all the symbols that are identified as a particular character.) For instance, if the allowable input characters are the alphabetics A through Z, then there will be an average character for an A, one for a B, etc. An unknown character is identified by comparing it to this set of average characters and determining that average character to which it most closely corresponds (the measure of correspondence will be defined below).

The unknown character is represented by the distribution of marks in two dimensions. An average character is likewise represented by a two-dimensional function. The value of this function at a point is the frequency of occurrence of a mark at that point computed over the character class of the average character. Fig. 1 shows an example of functions representing an average character and an unknown character.

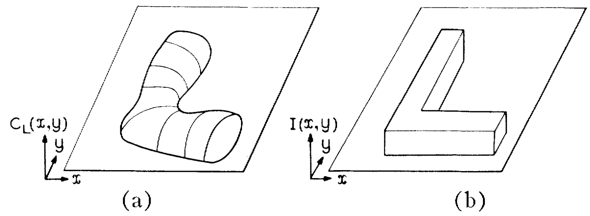

Fig. 1—Two-dimensional functions representing: (a) An average L. (b) An input (unknown) character (an L).

The measure of correspondence between an unknown character and a particular average character is the cross-correlation value between the two. An unknown character is identified with that character class represented by the average character with which the greatest cross-correlation value is obtained. Since this is a position-sensitive identification criterion, the unknown character must be shifted in two dimensions with respect to each average character. A cross-correlation function (a function of this two-dimensional shift) is computed between the unknown character and each average character. The maximum of each such function is chosen to represent the correspondence between the unknown character and that average character. The absolute maximum of these local maxima then forms the recognition criterion.

The above statements are formulized below:

The cross-correlation function between the unknown character and the jth average character as used here can be defined as[3]

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[1] K. R. Eldridge, F. J. Kamphoefner, and P. H. Werdt, "Automatic input for business data-processing systems," *Proc. EJCC*, New York, N. Y., December 10–12, 1956, pp. 69–73.

[2] W. K. Taylor, "Pattern recognition by means of automatic analog apparatus," *Proc. IEE*, vol. 106, pt. B, pp. 198–209; March, 1959.

[3] C. K. Chow, "An optimum character recognition system using decision functions," IRE Trans. on Electronic Computers, vol. EC-6, pp. 247–254; December, 1957.

$$\Phi_j(\sigma, \rho) = \frac{\int_x \int_y I(x+\sigma, y+\rho) C_j'(x, y) dxdy}{\left[\int_x \int_y I^2(x, y) dxdy \int_x \int_y C_j'^2(x, y) dxdy\right]^{1/2}}, \quad (1)$$

where

$C_j'(x, y)$ = the two-dimensional function representing the average character,

$I(x+\sigma, y+\rho)$ = the two-dimensional function representing the unknown (Input) character, shifted with respect to $C_j'(x, y)$ by distances σ, ρ in the x, y directions,

$\Phi_j(\sigma, \rho)$ = the cross-correlation function between $C_j'(x, y)$ and $I(x+\sigma, y+\rho)$, as a function of the two-dimensional shift, σ, ρ,

$\int_x\int_y(\ )dxdy$ = integral over the two-dimensional character field.

The integral $\int_x\int_y C_j'^2(x, y)\ dxdy$ is the norm of $C_j'(x, y)$. If $C_j'(x, y)$ is normalized by the square root of its norm, then the resulting function $C_j(x, y)$ is

$$C_j(x, y) = \frac{C_j'(x, y)}{\left[\int_x \int_y C_j'^2(x, y) dxdy\right]^{1/2}}, \quad (2)$$

and

$$\int_x \int_y C_j^2(x, y) dxdy = 1. \quad (3)$$

Then,

$$\Phi_j(\sigma, \rho) = \frac{\int_x \int_y I(x+\sigma, y+\rho) C_j(x, y) dxdy}{\left[\int_x \int_y I^2(x, y) dxdy\right]^{1/2}}. \quad (4)$$

For the remainder of this paper, the average character function, $C_j(x, y)$, will always be assumed to be normalized so that (3) holds.

Note that the norm of $I(x, y)$, $\int_x\int_y I^2(x, y)\ dxdy$, is common to all $\Phi_j(\sigma, \rho)$ for a particular input pattern; hence, neglecting it causes no reordering of the $\Phi_j(\sigma, \rho)$. Hence, maximizing the modified cross-correlation function $\Phi_j'(\sigma, \rho)$, given by

$$\Phi_j'(\sigma, \rho) = \int_x \int_y I(x+\sigma, y+\rho) C_j(x, y) dxdy, \quad (5)$$

is an equally valid recognition criterion. The true correlation value given by (4) will be used in the description of the simulation study of this method so that comparisons between recognition attempts can be easily made. However, the simplified form (5) will be used to advantage in the discussion of the optical implementation.

A modification to the $C_j(x, y)$ was studied. This is the addition of penalty areas to the average characters. A penalty area is an area of low probability to which a negative penalty weight is assigned. In each average character, the penalty weight for all penalty areas is a constant. It is arbitrarily chosen so that the integral of the penalty weight over all penalty areas in a given average character is unity.

Although the final machine using this method does not necessarily require quantization, the simulation of the method on a digital computer does. For purposes of simulation, an unknown character is represented by a 12×12 matrix of ones and zeroes. (This is both a spatial and a mark-intensity quantization.) A one corresponds to a mark in an element, a zero to no mark. The decision concerning the presence of a mark is based upon an appropriate threshold level. Each average character is also represented by a 12×12 matrix, with the value assigned to each element being proportional to the probability of occurrence of a mark in that element. The integrals in (3), (4), and (5) must then be replaced by the appropriate sums:

$$\Phi_j(\sigma, \rho) = \frac{\sum_m \sum_n I_{(m+\sigma),(n+\rho)} C_{jmn}}{\left[\sum_m \sum_n I_{mn}^2\right]^{1/2}}, \quad (6)$$

$$\Phi_{j'}(\sigma, \rho) = \sum_m \sum_n I_{(m+\sigma),(n+\rho)} C_{jmn}, \quad (7)$$

where

$$\sum_m \sum_n C_{jmn}^2 = 1. \quad (8)$$

The notation is the same as that used previously, except that the discrete subscripts m, n replace the continuous variables x, y. σ and ρ are also discrete in this case. Note that $\Phi_j'(\sigma, \rho)$ in the quantized case above is simply the sum of the weights of the marked elements of C_j, since each I_{mn} can only be zero or one. Likewise, the norm of I, $\sum_m \sum_n I_{mn}^2$, is simply the sum of the marked elements in the matrix representing the input character.

A simple example will illustrate the mechanics of this method. Assume that the unknown and average characters are represented by 3×3 matrices. Fig. 2(a) and (b) shows hypothetical unnormalized and normalized average characters (hereafter called probability matrices) for a C and an 0. Penalty weights have been added to areas of near-zero probability. An input character (an 0) is shown in Fig. 2(c). Shifting is not performed in this simple example since the optimum positions are obvious. The pertinent modified correlation values are shown, the maximum of which clearly identifies the input character properly.

0.8	0.9	0.8
0.9	0.3	−1
0.8	0.9	0.8

AN AVERAGE C

0.8	0.9	0.8
0.9	0.3	0.9
0.8	0.9	0.8

AN AVERAGE O

(a)

0.36	0.40	0.36
0.40	0.13	−1
0.36	0.40	0.36

NORMALIZED AVERAGE C

0.33	0.37	0.33
0.37	0.12	0.37
0.33	0.37	0.33

NORMALIZED AVERAGE O

(b)

1	1	1
1	0	1
1	1	1

AN INPUT O

$\phi'_C = 1.64$

$\phi'_O = 2.80$

(c)

Fig. 2—A recognition example.

Simulation

This character recognition method was studied by simulation on the IBM 704 computer. As previously discussed, the characters were represented by 12×12 matrices of one's and zero's. This degree of quantization introduced a quantizing error affecting the results, but was necessary to maintain reasonable computer time.

The method was applied to two different sets of data: a set of hand-printed alphanumeric characters and a set of machine-printed numbers. The hand printing consisted of 1800 characters (50 alphabets of the 36 alphanumeric characters) printed by 50 different people. This printing was somewhat constrained by requiring the writer to print on one-quarter inch quadruled paper, asking him to print neatly and at a size approximating the ruled boxes on the paper.

The source of the machine printing was an IBM 407 line printer. 1000 numbers were studied representing 100 samples of each of the ten numerals. These were taken from 80 different type wheels.

The samples were scanned and converted to matrix form by the use of the generalized scanner,[4] an optical scanner that can be programmed to generate any type of scan within its resolution capabilities. The scanner output is a magnetic tape compatible with the IBM 704 computer.

The primary value of the study of hand printing was the sensitivity of the recognition results to various parameters (such as various methods of centering, penalty area variations, etc.). Since the hand-printed data was not as "handicapped" by high percentage recognition as the machine-printed data (recognition rates for the former were in the order of 60–80 per cent), this data furnished a sensitive test to evaluate variations in the recognition methods.

The particular parameter ranges which were determined to be best were then applied to the recognition of machine-printed characters.

Parameters of Investigation

One parameter of the simulation study was the method of centering. Two methods were investigated: centering by center of gravity alignment, and centering by maximizing the cross-correlation value as a function of position (centering by shifting). In the former, the center of gravity of the input pattern is aligned with the center of gravity of the probability matrix (average character). In the latter method, the input pattern is shifted in two dimensions with respect to each probability matrix, and the cross-correlation value is computed for each position. Hence, the correlation obtained with a particular probability matrix is a function of input pattern position. The maximum value of this correlation function is used as the measure of fit between the input pattern and that probability matrix. Although centering by shifting seems to be intuitively better, center-of-gravity centering has some advantages. For one, it requires much less computer time for simulation. In addition, there was the possibility that centering in this manner would eliminate the tendency of a character to find a false maximum correlation, when compared to a probability matrix other than its own, by finding some opportune misalignment.

A second parameter involved the question of penalty weights. Penalty weights are negative numbers assigned to elements of low probability. Hence if an unknown character falls in a region of low probability with respect to a particular probability matrix, then the corresponding cross-correlation value is reduced, or "penalized." Penalty weights then have the possibility of increasing the discrimination between characters. In this study, a penalty threshold level was chosen so that any elements with a probability less than the penalty threshold would be assigned the penalty weight. All penalty elements were assigned identical weights, and these weights in each matrix were arbitrarily normalized so that the sum of the squares of the weights was unity (the same normalized value of the matrix) as previously discussed. The value of the penalty threshold level was varied to determine the effect on error rate. The effect of nonuniform penalty weights and the effect of other normalizing criteria were not studied.

The third parameter studied was that of rejection criteria. Here we are interested in setting certain criteria for the final cross-correlation values in order that the recognition be acceptable. If the recognition is not acceptable, the character is rejected as being unreadable. Through the use of rejection criteria, the undetected error rate (substitutional errors) can be made as small as desired by making the rejection rate as large as necessary. The particular rejection criteria considered required that the maximum correlation value exceed a particular threshold level and, further, that it be greater than the next highest correlation value by a prescribed discrimination level.

In summary, then, the parameters of this study included:

1) Centering methods.
2) Penalty areas.
3) Rejection criteria.

[4] W. H. Highleyman and L. A. Kamentsky, "A generalized scanner for pattern- and character-recognition studies," *Proc. WJCC*, San Francisco, Calif., March 3–5, 1959; pp. 291–294.

Construction of Probability Matrices

An unnormalized probability matrix for a particular character is constructed by determining the probability of occurrence of a mark in each of the elements of the matrix when the input pattern is that character. In this study, 50 samples of the pertinent character were used to construct each probability matrix. Different probability matrices were, of course, used for the machine printing and for the hand printing.

Since there is no mechanism for properly centering the characters in the matrix scanning process, care must be taken to assure that they are properly centered before constructing the probability matrix. First, though, one must decide just what constitutes proper centering. Since all of the recognition methods concerned maximize a function which is monotonically increasing with the cross-correlation function, it seems reasonable to define proper centering of an input pattern with respect to the probability matrix as that position which maximizes the cross-correlation function between the two.

However, in initially constructing the probability matrices, there exist no such matrices which can be used to center the patterns. Therefore, the process of construction must be an iterative one. Considering the case of a particular character, the first step is to construct a probability matrix for that character from the unshifted sample members. Then the cross-correlation function (as a function of two-dimensional shift of a maximum of ± 5 elements in each direction, or 121 positions) is computed for each sample member compared with the first probability matrix, and its optimum position with respect to the first probability matrix is determined by the maximum of the correlation function. When all of the optimum positions of the sample members have been found, they are shifted to these positions, and a new probability matrix is constructed. This process is repeated between the sample members and the probability matrix until the elements of the probability matrix converge to their final values. The IBM 704 computer was utilized to carry out the iterations.

It seems reasonable that a test of convergence of the elements of such a matrix might be the auto-correlation value (the sum of the squares of the probabilities) of that matrix. That is, the final probability matrix is that matrix which maximizes the cross-correlation values of all of the component matrices (the sample members) with itself; therefore, one might expect that this also maximizes the auto-correlation value of the probability matrix.

The auto-correlation value was used to test the convergence of this iteration process. It was indeed a valid test, as most of the matrix elements for the hand printing converged after seven or eight iterations, and the machine printing matrix elements converged after three or four iterations. Each iteration took 30 seconds on the IBM 704 computer.

Fig. 3 shows some typical convergence curves for some of the characters. Figs. 4 and 5 show examples of the first and final unnormalized probability matrices for a hand-printed character and for a machine-printed character. The numbers in these matrices are the actual number of sample members which contained a mark in that element; division by 50 yields the probability of occurrence of a mark.

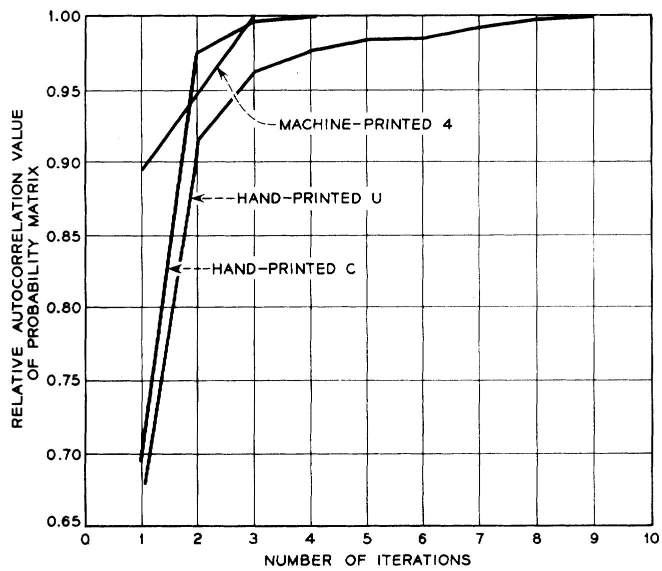


Fig. 3—Convergence curves for probability matrix iteration.

0	0	0	0	1	2	3	3	2	1	1	0
0	0	2	2	8	11	12	12	6	1	1	0
0	1	7	14	25	32	32	33	21	6	3	1
0	6	20	21	26	16	11	16	13	6	2	0
3	15	25	22	11	6	2	3	4	3	2	0
7	25	24	12	2	1	1	0	0	0	0	0
5	20	30	12	1	1	1	0	3	2	2	0
6	16	19	12	15	15	14	17	13	6	3	1
4	9	14	15	17	15	17	16	12	4	3	0
2	3	7	7	13	10	8	6	6	5	0	0
2	0	1	2	4	4	5	4	1	1	0	0
1	0	1	1	3	1	1	0	0	1	0	0

(a)

0	0	0	0	1	0	1	0	2	0	1	0
0	0	1	1	1	9	18	18	9	4	3	2
0	0	1	7	37	44	39	32	23	6	2	2
0	0	18	46	30	6	8	10	9	8	2	1
0	8	47	20	0	1	3	1	2	4	0	0
0	27	41	3	0	1	0	0	0	0	0	0
0	28	30	2	1	1	3	2	2	1	2	0
0	16	26	15	10	9	11	8	7	7	4	1
0	5	16	23	25	25	24	23	16	9	4	0
0	1	2	10	11	10	10	8	6	2	0	0
0	0	0	2	1	2	2	1	2	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

(b)

Fig. 4—(a) Initial probability matrix for hand-printed *C*. (b) Final probability matrix for hand-printed *C*.

0	0	0	0	32	32	0	0	0	0	0	0
0	0	3	25	39	24	3	0	0	0	0	0
0	1	13	38	33	8	2	8	1	0	0	0
0	5	36	41	12	2	15	24	8	0	0	0
3	19	43	31	2	0	21	43	13	0	0	0
22	44	49	31	21	19	36	48	37	21	11	0
28	47	49	45	47	46	48	50	47	46	32	0
7	19	17	18	24	19	37	49	34	20	5	0
0	1	0	2	2	1	19	46	23	1	0	0
0	0	0	0	0	0	3	15	6	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

(a)

0	0	0	0	30	38	1	0	0	0	0	0
0	0	0	25	49	22	0	0	0	0	0	0
0	0	9	43	40	1	0	10	2	0	0	0
0	1	39	50	6	0	9	30	11	0	0	0
2	20	50	30	0	0	13	49	14	0	0	0
24	49	50	26	22	21	40	50	36	22	16	0
35	50	50	50	50	50	50	50	50	50	33	0
6	13	11	16	20	16	32	50	37	13	1	0
0	0	0	0	0	0	13	48	25	0	0	0
0	0	0	0	0	0	3	13	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

(b)

Fig. 5—(a) Initial probability matrix for machine-printed 4. (b) Final probability matrix for machine-printed 4.

Recognition of Hand Printing

As mentioned previously, the primary purpose of studying hand-printed characters was to determine the effects of the various parameters. It was found early in the study that the use of all of the 1800 hand-printed characters for all of the tests was prohibited by the computing time required. For instance, a recognition trial using these characters in which centering is accomplished by shifting the input pattern a maximum of two elements in each direction required six hours of 704 time. Therefore, the determination of the effect of these parameters was deduced from just the hand-printed numbers (the same 500 characters which were used to construct the probability matrices). To process this subset of the hand-printed characters required about fifty minutes of computer time under the above conditions. The optimum values thus found were then applied to the total set of hand-printed characters and to the machine-printed numbers.

The graph of Fig. 6 presents the error rates as a function of the method of centering and the penalty criteria for the hand-printed numbers.

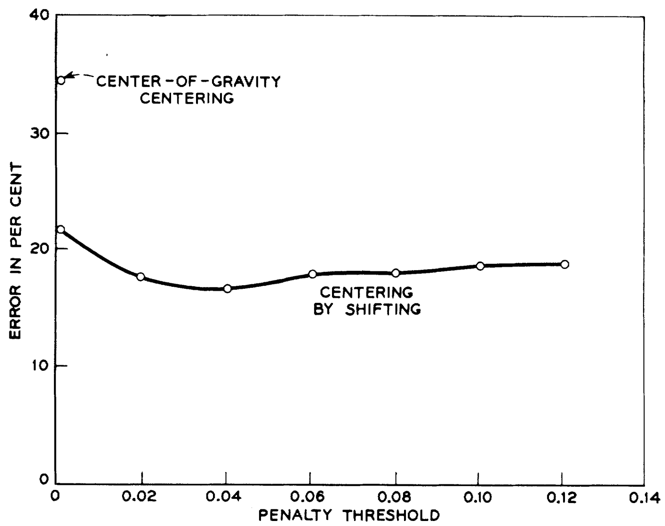


Fig. 6—Error rate for various parameters using hand-printed numbers (500 samples).

Penalty Criteria

The abcissa of Fig. 6 represents the penalty threshold used in the various tests. Each element with a probability *less than* the penalty threshold is assigned a penalty weight. Hence, the ordinate axis of the graph corresponds to error rates in which penalty areas were not used. As discussed earlier, these penalty weights are negative numbers which are constant in each matrix, and which are adjusted so that the sum of their squares in a particular matrix is unity.

In Fig. 6 the effect is shown of the penalty threshold on the per cent error. Note that the error is a minimum for a penalty threshold of 0.04.

Centering

The two methods of centering which were studied were described earlier. The particulars of centering by shifting warrant comment. In this case, to minimize simulation time, the input pattern was first roughly aligned with a probability matrix by using center-of-gravity alignment. Then the input pattern was shifted a maximum of two elements in all horizontal and vertical directions (25 positions). The maximum value of of the correlation function (a function of position) was chosen to represent the degree of match between the input pattern and that probability matrix to which it was being compared.

The two methods of centering were studied for the case of zero penalty threshold. The results are shown in Fig. 6, in which it is seen that centering by shifting is significantly better. Hence, whatever advantages center-of-gravity centering might have had (as discussed earlier) were not borne out by these results.

Rejection Criteria

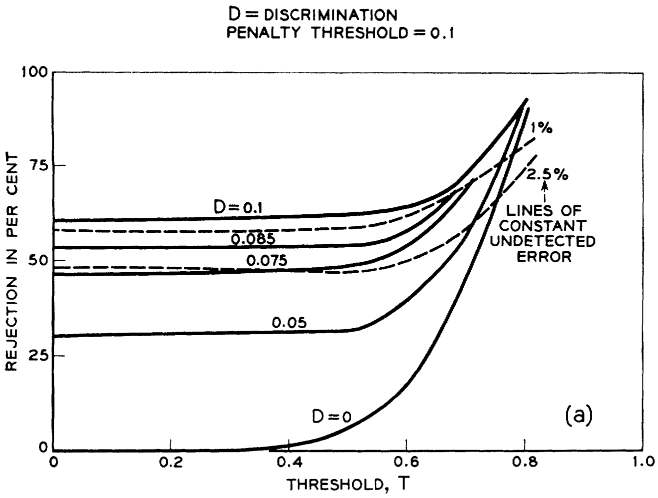
The two rejection criteria studied, in review, are criteria applied to the resulting correlation values which determine an acceptable recognition. One criterion is a threshold level below which a score is rejected. The other is a discrimination level which requires that the top score and the next highest score be separated by a certain amount. The effect of various recognition criteria on hand-printed numbers was studied in detail for the case of a penalty threshold of 0.1 and centering by shifting.

Fig. 7(a) shows the dependence of the over-all rejection rate on the threshold level (T) and the discrimination level (D). Fig. 7(b) illustrates the dependence of the undetected (substitutional) error rate on the rejection parameters (the per cent undetected error rate is the per cent of the whole sample).

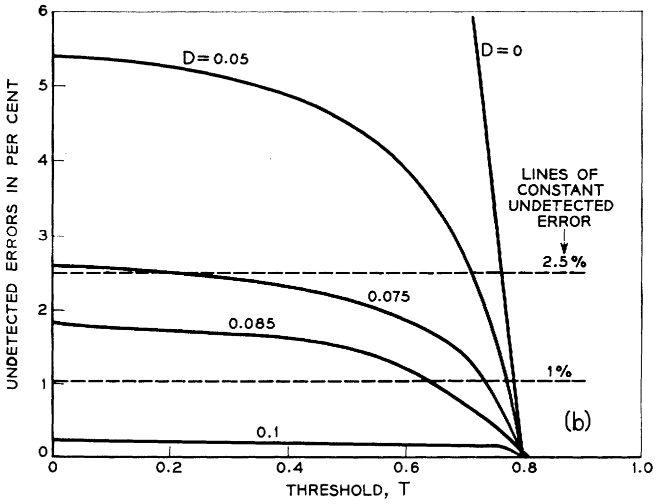
It is of interest to plot the loci of constant undetected error rate on these graphs so that the rejection rate which is required to achieve a desired maximum undetected error rate can be discovered. In Fig. 7(b), these loci are simply horizontal lines. Some loci for particular error rates are shown dotted. The intersections of these loci with lines of constant D can be used to plot similar loci on the graph of Fig. 7(a), where they are shown again with dotted lines.

The interesting result of this construction, as seen from Fig. 7(a), is that the minimum rejection rate for a prescribed error rate occurs for $T=0$ and D a particular value. That is, for this particular set of samples, the threshold level is meaningless as a criterion for rejection. Evidently, characters with very low maximum correlation values should still be accepted as long as the difference between the highest and next highest correlation value is sufficient.

Using, then, the optimum values of the recognition



(a)



(b)

Fig. 7—Per cent rejection and per cent undetected errors as a function of the rejection parameters. D = discrimination. Penalty threshold = 0.1.

criteria (*i.e.*, $T=0$), the dependence of undetected error rate on rejection rate can be determined. This relation, for a penalty threshold of 0.1, is shown in the graph of Fig. 8 by the points enclosed in circles. Note that the resulting curve is approximately a straight line in the region considered.

Assuming then a linear dependence between error rate and rejection rate, similar curves for other penalty thresholds were determined. Shown in Fig. 8 as a heavy line are those particular values which give a minimum rejection rate for a particular error rate, and also the curve for a penalty threshold of 0.04. It was this latter value that yielded the lowest error rate before rejection criteria were applied (see Fig. 6).

Some interesting points can be noted from Fig. 8:

1) The relation between undetected error rate and rejection rate for a given penalty threshold is approximately linear in regions of low error rate, as previously noted.

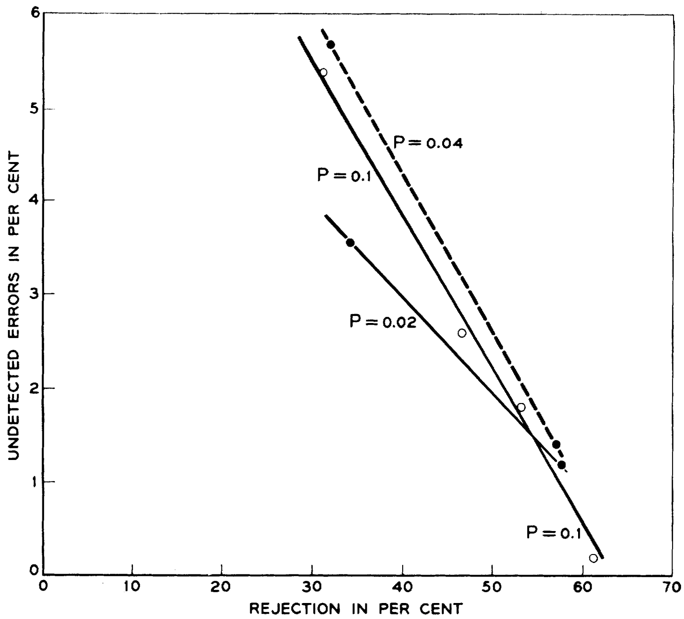


Fig. 8—Undetected error rate vs rejection rate for hand-printed numbers. P = penalty threshold.

2) The penalty threshold value for which the lowest real error rate is attained is not necessarily optimum when one considers rejection criteria.
3) The particular penalty threshold which is optimum when rejection criteria are used is a function of the undetected error rate desired.

Examples of Hand-Printed Samples

Although the primary purpose of studying hand printing was to ascertain the effects of certain parameters on the recognition ability of this method, it is of interest to determine just how well the optimum parameters would do on hand printing. Consequently, this method, with centering by shifting and a penalty threshold of 0.04, was applied to the hand-printed alphanumeric alphabet of 1800 samples. No rejection criteria were applied. The total recognition rate was 77.2 per cent.

In Fig. 9 are shown some of the actual input data used. Shown in Fig. 10(a) are some matrix forms of high quality and degraded characters which were read correctly. In Fig. 10(b) are matrix forms of some characters read incorrectly. The entries beneath the matrices in Fig. 10(a) show the first and second choices and their correlation values (the first choices are all correct). Below the matrices in Fig. 10(b) are the first choices and their correlation values, as well as the actual identity of the characters with their associated correlation values.

A note of caution is necessary here. These results for the hand printing are based on the same data which were used to determine the probability matrices. It is quite doubtful that 50 alphabets comprise a large sample of hand-printed characters. Hence, one would expect a

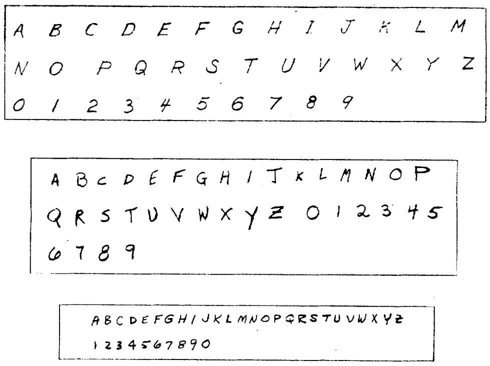


Fig. 9—Some samples of the hand-printed data.

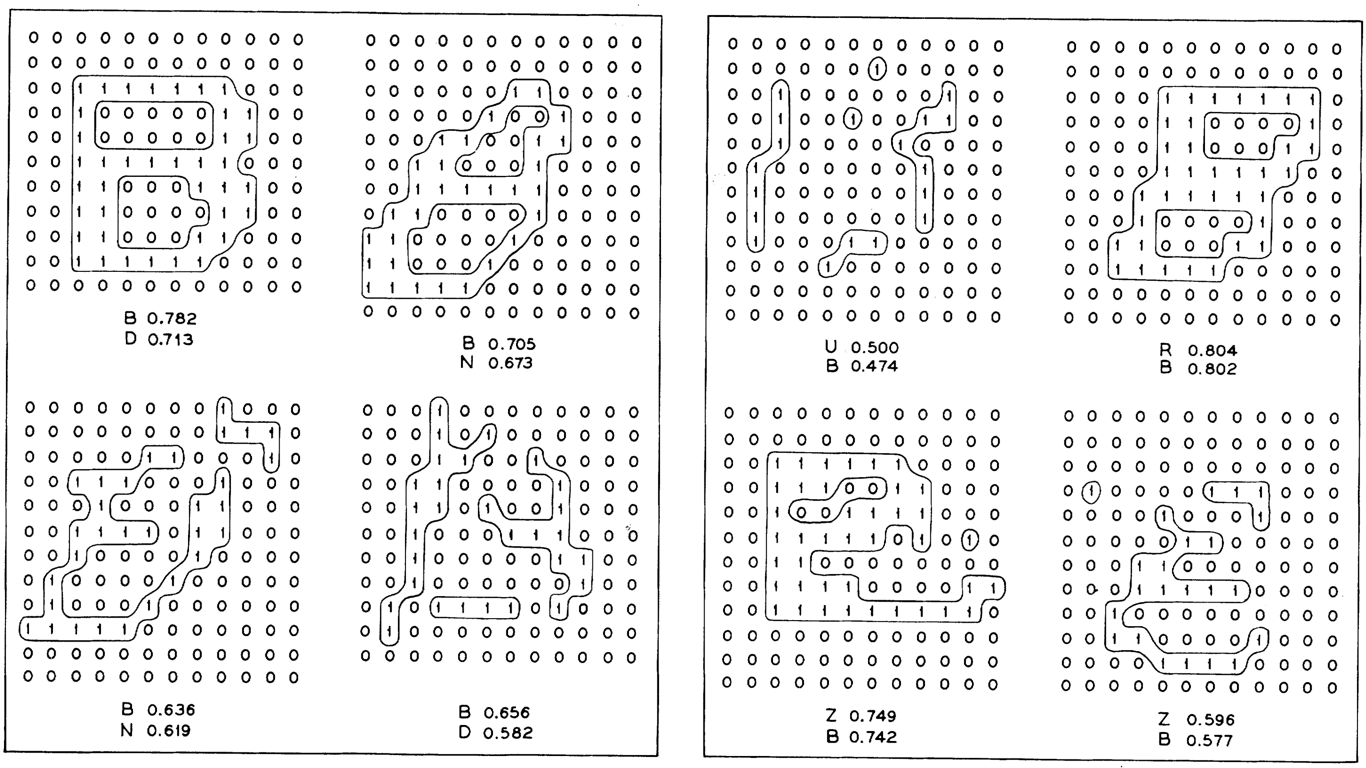


Fig. 10—(a) Hand-printed *B*'s recognized correctly. (b) Hand-printed *B*'s recognized incorrectly.

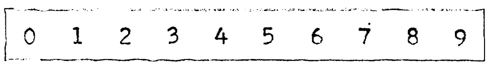


Fig. 11—Sample of IBM 407 printing.

significantly lower recognition rate for input characters other than the original data. However, it is possible that sufficient writing constraints on the originator may exist which would yield a usable recognition rate with this method. A finer quantization might also give some improvement.

Machine Printing

Parameters: As a consequence of the above investigation, recognition with centering by shifting was applied to the 1000 machine-printed numbers, a sample of which is shown in Fig. 11. The penalty threshold was varied from 0 to 0.06 to find an optimum value, since there was no optimum value clearly indicated by the previous investigation. The minimum rejection criteria required to detect all errors was applied. In this case, the number of errors was so small that a perusal of the data showed that a rejection threshold T of zero was still optimum.

Results

The results of using the above parameters are shown in Fig. 12. The discrimination level D ($T=0$) required at each point is shown. It is clear from this graph that

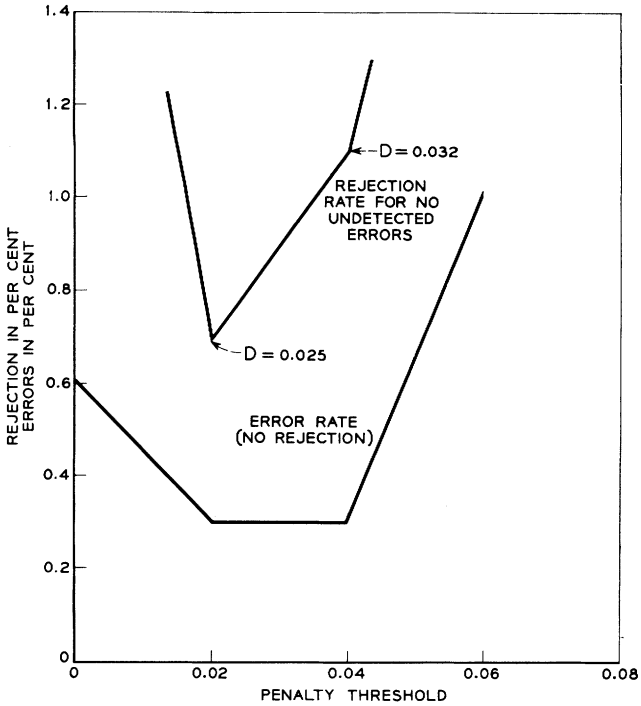


Fig. 12—Error and rejection rates for machine-printed numbers. D = discrimination level.

the optimum penalty threshold is 0.02, for which a rejection rate of 0.7 per cent guarantees no errors. Without rejection, the error rate is 0.3 per cent at this point.

Of the set of 1000 machine-printed members studied, 500 were used to construct the probability matrices. Since one might expect these characters to do better in the recognition process than the remaining 500, it is of interest to compare the results of the two sample subsets. For the subset used to construct the probability matrices, the per cent error was 0.2 per cent and the per cent rejection was 0.8 per cent. For the remaining samples, the per cent error was 0.4 per cent and the per cent rejection was 0.6 per cent. Since there is little difference between these results, the results of either subset or of the complete set should be valid.

Error Analysis

Because of the small number of errors in the optimum case for machine printing, each one can be examined in detail. The matrix forms of some normal characters are shown in Fig. 13. In Figs. 14 and 15 are shown the seven rejected numbers, the ones in Fig. 14 being the ones which were incorrectly recognized. Below these matrix forms are correlation values similar to those of Fig. 10.

The errors in Figs. 14(c) and 15) are explainable as centering or quantizing errors. The reasons for the errors shown in Fig. 14(a) and (b) are not clearly understood.

Fig. 16 (next page) shows some degraded characters which were recognized correctly, along with the first and second choice correlation values.

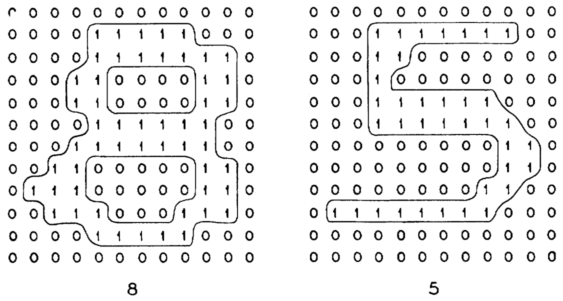


Fig. 13—Examples of normal machine-printed characters.

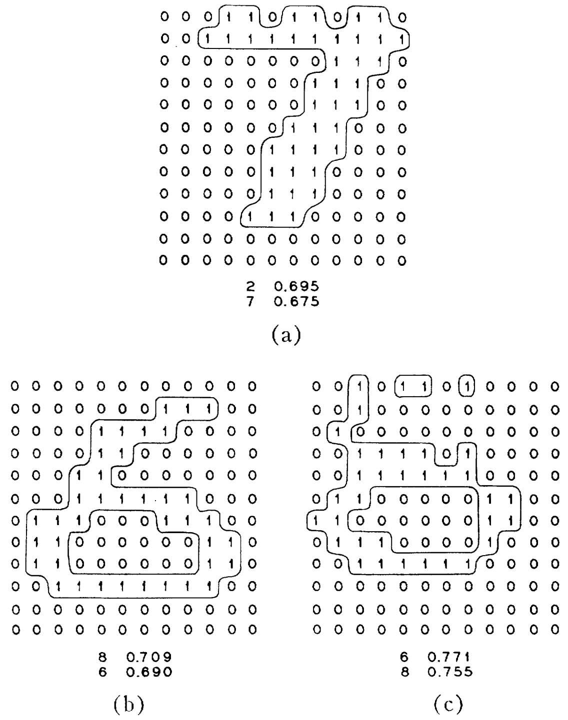


Fig. 14—Machine-printed numbers recognized incorrectly.

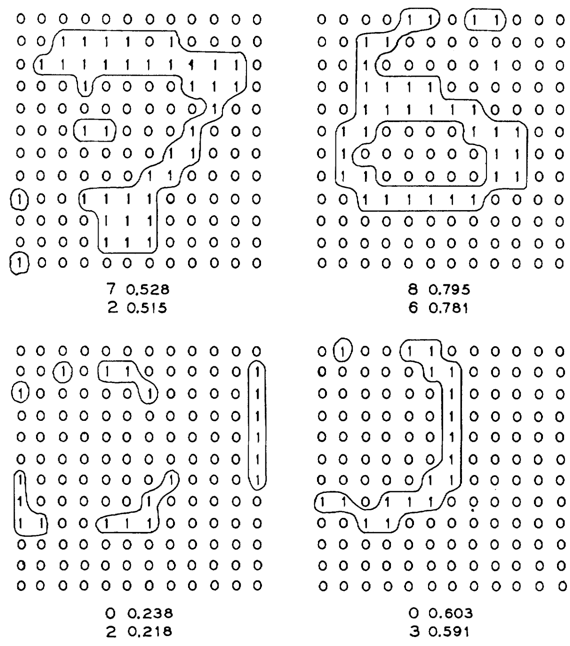


Fig. 15—Machine-printed numbers recognized correctly but rejected along with the numbers of Fig. 14.

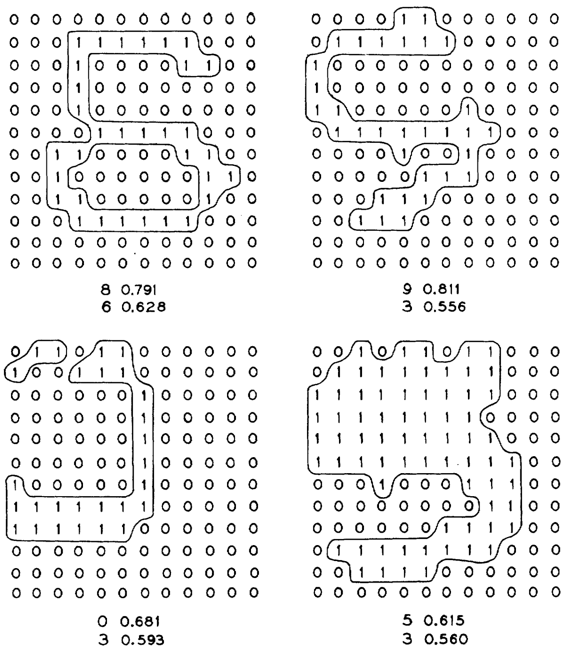


Fig. 16—Some degraded machine-printed numbers recognized correctly.

Summary of Results

The important results of this study are summarized in Table I. It seems that this method should be applicable to the reading of machine-printed characters, and that, with proper engineering effort, the error rate and rejection rate could be traded for one another and could be made quite small. The economical implementation which can be obtained by using analog techniques is described in the next section.

TABLE I

SUMMARY OF MAJOR RECOGNITION RESULTS

*Hand Printing (Alphanumerics)**	
Per Cent Recognition, Alphanumerics	77.2
Per Cent Recognition, Numbers	83.0
Machine Printing (Numerics)†	
Per Cent Recognition	99.7
Per Cent Rejection for no Undetected Errors	0.7

* Results based on the same data used to determine the probability matrices (1800 characters total).

† Results based partly on data used to determine the probability matrices, and partly on additional data (1000 characters total).

AN OPTICAL IMPLEMENTATION

Optical Correlation

The character recognition method described in this paper can be economically implemented by electro-optical techniques. The implementation consists essentially of a transparency-photomultiplier combination for each character. The transparency represents the average character. When the image of the unknown character is focused upon the transparency, the transmitted light, measured by the photomultiplier, is a function of the desired cross-correlation value. (This is similar to comparison techniques described by Davis and Norwine[5] and by Bozeman.[6])

Consider a piece of film in which the transparency at each point is proportional to the probability of occurrence of a mark at that point for a particular character. That is, the transparency of the film represents an average character as previously described. Let some input character be focused upon the transparency (Fig. 17). Then the light transmitted through the transparency at a point is a function of the product of the reflectance of the paper and the transmittance of the film at that point. That is, let

$a(x, y)$ = absorption distribution of input pattern,
$r(x, y)$ = reflection distribution of input pattern $= 1 - a(x, y)$,
$t(x, y)$ = transmission distribution of film,
$i(x, y)$ = light intensity transmitted through film.

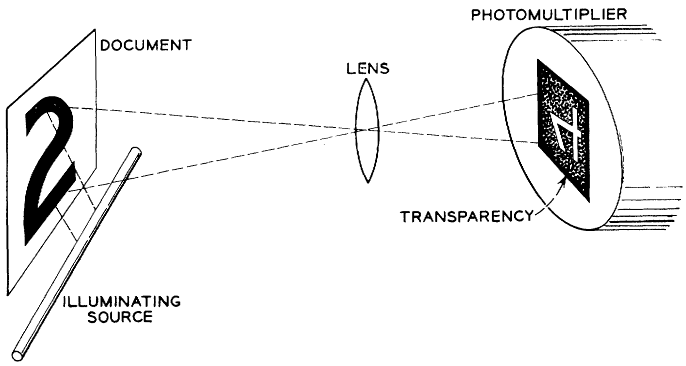


Fig. 17—An optical correlation channel.

Then,

$$i(x, y) \sim r(x, y)t(x, y). \tag{9}$$

If the input pattern is shifted an amount $x=\sigma$, $y=\rho$ with respect to the film then the total light flux, $I(\sigma, \rho)$, transmitted through the film is

$$I(\sigma, \rho) \sim \iint r(x+\sigma, y+\rho)t(x, y)dxdy \tag{10}$$

or

$$I(\sigma, \rho) \sim \iint t(x, y)dxdy$$

[5] K. H. Davis and A. C. Norwine, U. S. Patent No. 2,646,465; July 21, 1953.

[6] J. W. Bozeman, U. S. Patent No. 2,898,576; August 4, 1959.

$$-\iint a(x+\sigma, y+\rho)t(x, y)dxdy, \quad (9)$$

$$I(\sigma, \rho) \sim T - \Phi'(\sigma, \rho), \quad (10)$$

where

T = a constant, different for each character (actually the sum of the probabilities of the probability matrix).

$\Phi'(\sigma, \rho)$ = the modified cross-correlation function between $a(x, y)$ and $t(x, y)$ as a function of the two-dimensional shift σ, p, analogous to (5).

Since $a(x, y)$ represents the mark distribution of the input pattern and $t(x, y)$ represents a probability matrix, we are interested in $\Phi'(\sigma, \rho)$, the cross-correlation function between the input pattern and a given probability matrix.

$I(\sigma, \rho)$ can be measured by a photomultiplier (Fig. 17) which views the entire field of the film. Subtracting T from the output of the photomultiplier will then cause the output to be proportional to the cross-correlation function Φ. This adjustment is easily made by placing a white piece of paper in the field of view. Then $I \sim T$, $a(x, y)$ being arbitrarily taken as zero for white paper. The compensating voltage is then adjusted to make the photocell output zero, making $I \sim \Phi'$ thereafter.

Another normalization is required. It is important that the probability matrices be normalized to some common value, as discussed previously, such that

$$\iint t^2(x, y)dxdy = N.$$

This normalization is made (once the previous compensation for T has been made) by adjusting the gain of the photochannel.

Penalty Areas

One problem which appears is that of handling penalty areas. Penalty areas are regions of low probability in the probability matrix to which negative weights are assigned. Obviously, one cannot obtain a negative transmittance with a piece of film.

However, note that a constant can be added to every element of every probability matrix in the system. If C is the value of this constant, and P the number of marked elements in the input pattern, then this modification simply causes a constant (PC) to be added to every cross-correlation value. The ordering of the cross-correlation values is not affected and the recognition is still valid.

Therefore, a positive C can be chosen so that its magnitude is equal to that of the greatest penalty weight. The elements of all probability matrices are then assured to be positive after the addition of C.

A Film Correlator

In Fig. 18 a transparency which might be used in an optical correlator is shown. It represents the probability matrix for a machine-printed "2" (from the IBM 704 line printer) with penalty weights in areas of zero probability.

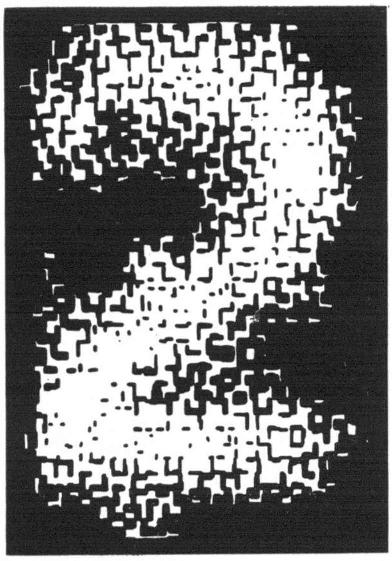


Fig. 18—A transparency for an optical correlator.

It was constructed by extending a 12×12 probability matrix (determined by the IBM 704 computer) to a 36×36 matrix by interpolation. A constant was added to each element so that the penalty areas all had zero weight. Then each element in the matrix was filled in with ink so that the proportion of area left unfilled was the ratio of the weight of that element to the largest weight in the matrix. Hence, unit probability elements are completely open, whereas zero probability elements are filled in completely. Note that, although this example indicates quantization, the quantization can be made arbitrarily small at the expense of additional computer time.

A Recognition System

The basic components of a character recognition system using optical correlation are shown in Fig. 19 for the case of four channels. The extension to n channels is obvious. The combination of the document motion and rotating mirror creates the required two-dimensional shift of the input pattern with respect to each probability matrix. The optical correlator has been discussed above. Each feeds an analog storage device, which, in turn, drives the comparator. The analog storage consists simply of a diode fed capacitor. A possible basic form of the comparator is shown in Fig. 19; briefly only that transistor with the largest base voltage will be conducting when gated. Note that the

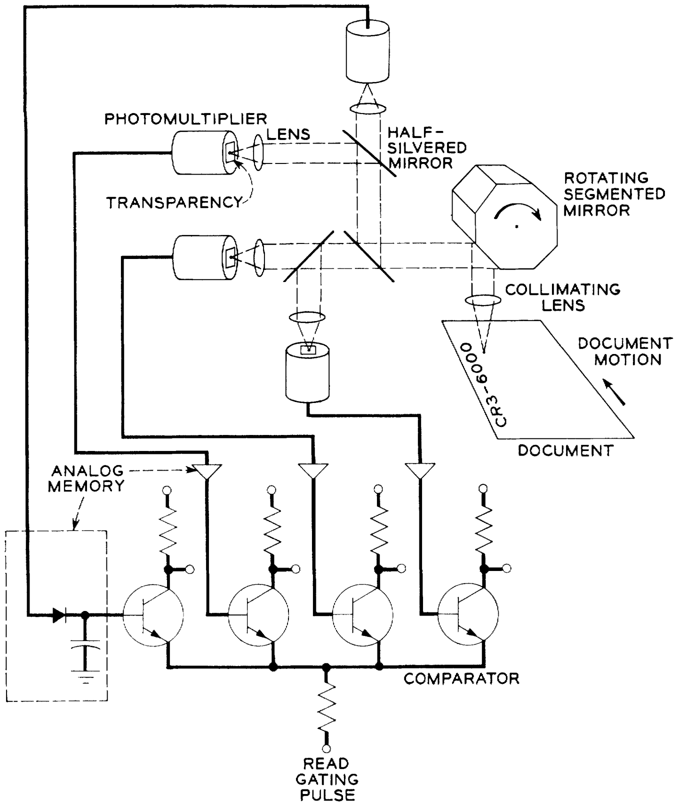


Fig. 19—A character-recognition system using optical correlation.

optics are arranged so that each light path goes through the same number of half-silvered mirrors and lenses to equalize loss, although this is not a fundamental requirement. Additional circuitry is of course required for the various control functions, such as timing, rejection decisions, and resetting the analog store.

Conclusion

A character recognition method capable of an economical analog implementation using optical techniques has been proposed. This method has been simulated on the IBM 704 computer and has been shown to be applicable to machine printing and perhaps to constrained hand printing. The author feels that this recognition method exemplifies some of the many advantages (such as low cost and lack of quantizing error) that can be gained by considering analog implementation in the construction of recognition and allied equipment.

Acknowledgment

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The Hall-Effect Analog Multiplier*

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***Summary*—The application of the Hall effect to a general-purpose four-quadrant multiplier is discussed. Circuit diagrams for the transistor amplifiers are given. An evaluation of the experimental results is given for a breadboard model of the multiplier. Static accuracies on the order of 1 per cent to 3 per cent are obtained for the Hall channel and the magnetic channel, respectively. Bandwidths of 25 kc and 1.3 kc are achieved for the Hall channel and the magnetic channel, respectively.**

Introduction

THIS PAPER discusses an analog multiplier which was constructed using an indium arsenide Hall-effect element as the basic multiplying device. As is widely known today, a direct means of analog multiplication is obtained by subjecting the charge carriers in a current-carrying semiconductor or conductor to the action of a magnetic field.[1–8] The voltage which is developed in the material (the Hall voltage) is in a direction mutually perpendicular to the

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